

Corrected

# International Islamic University Chittagong (IIUC)

Department of Electronic and Telecommunications Engineering

Final Examination

Program: B.Sc. (Engg.)

Course Code: Math-1107

Semester: Spring-2018

Course Title: Math-I

(Differential calculus)

Time: 2 hours 30 minutes

Full Marks: 50

## Group- A

[Answer *any two* sets of the following questions]

1. (a) State and prove Euler's theorem on homogeneous function 04  
(b) If  $u = \tan^{-1} \frac{x^3+y^3}{x+y}$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$  03  
(c) If  $v = (x^2 + y^2 + z^2)^{-1/2}$  show that  $x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} + z \frac{\partial v}{\partial z} = -v$  03
2. (a) Prove that  $\sqrt{3} \sin x + 3 \cos x$  has a ~~minimum~~ <sup>maximum</sup> value for  $x = \frac{\pi}{6}$  04  
(b) Discuss the maximum and minimum of the function  $f(x) = x^3 - 6x^2 + 9x + 5$  03  
(c) If  $u = x^2(y-z) + y^2(z-x) + z^2(x-y)$  show that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$  03
3. Integrate any four of the following: 10  
Evaluate any four of the following: (i)  $\int \sin^{-1} \frac{x}{a+x} dx$  (ii)  $\int \frac{dx}{(2+x)\sqrt{1+x}}$   
(iii)  $\int \frac{dx}{4+5\cos x}$  (iv)  $\int \frac{dx}{x(x+1)(x+5)}$  (v)  $\int \sqrt{\frac{a+x}{a-x}} dx$  (vi)  $\int \cos 2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} dx$

## Group- B

[Answer *any three* sets of the following questions]

4. Evaluate any four of the following differential integrals 10
  - (a)  $\int_1^2 \frac{dx}{\sqrt{(x-1)(2-x)}}$
  - (b)  $\int_0^\pi \frac{dx}{3+2\sin x + \cos x}$
  - (c)  $\int_0^1 \frac{dx}{\sqrt{2x-x^2}}$
  - (d)  $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} dx$
  - (e)  $\int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx$
5. (a) Find the value of  $\Gamma(\frac{1}{2})$  03  
(b) Show that  $\beta(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$  04

(c) • Show that  $\int_0^{\pi/2} \sqrt{\cot \theta} \, d\theta = \frac{1}{2} \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right)$

03

05

6. (a) Evaluate  $\int_a^b x^2 \, dx$  as the limit of a sum.

05

(b) Evaluate  $\lim_{n \rightarrow \infty} \left( \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+(n-1)} \right)$

05

05

7. (a) Show that the area between the parabola  $y^2 = 4x$  and the straight line  $y = 2x - 4$  is 9

(b) Find the length of the perimeter of the circle  $x^2 + y^2 = a^2$