International Islamic University Chittagong
Department of Computer Science & Engineering
B. Sc. in CSE
Semester Final Examination, Autumn 2018
Course Code: *CSE* 1223 Course Title: Discrete Mathematics
Total marks: ***50*** Time: 2 hours and 30 minutes

[Answer any ***two*** questions from each **Group-A** and **Group-B;**
Separate answer script must be used for Group-A and Group-B;
Figures in the right hand margin indicate full marks.]

Group-A

1. a) Prove that there are infinitely many primes. 3
   b) Express gcd(17, 29) as a linear combination of 17 and 29. Show the steps. 3
   c) Using the concept of Chinese reminder theorem, find a solution to the system of congruences $x \equiv 2 \pmod 3$, $x \equiv 1 \pmod 4$ and $x \equiv 3 \pmod 5$. 4

2. a) Show that square of an even number is also an even number using direct proof. 2
   b) Prove that $1^2 + 3^2 + 5^2 + \ldots + (2n+1)^2 = (n+1)(2n+1)(2n+3)/3$. 3
   c) What do you mean by mathematical induction? Why mathematical induction is valid? 2
   d) Prove that "every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps" using Strong Induction. 3

3. a) With these hypotheses: 3
   "It is not sunny this afternoon and it is colder than yesterday."
   "We will go swimming only if it is sunny."
   "If we do not go swimming, then we will take a canoe trip."
   "If we take a canoe trip, then we will be home by sunset."
   Using the inference rules, construct a valid argument for the conclusion:
   "We will be home by sunset."
   b) What are the differences between direct and indirect proof? Elucidate using some example. 4
   c) Show that if a number **n** does not have a prime divisor less than $\sqrt{n}$, then the number **n** is a prime. 3

Group-B

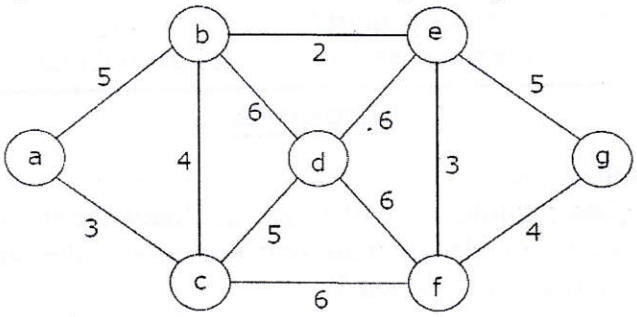
4. a) What is the minimum number of students required in a Discrete Mathematics class to be sure that at least eight will receive the same grade, if there are four possible grades, namely A+, A, A- and F. 2
   b) How many different strings can be made by reordering the letters of the word "SUCCESS"? 2
   c) Each user on a computer system has a password, which is five to six characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there? 3
   d) What do you mean by derangement? List all the derangements of {9, 55, 80,95} 3

5. a) Define the following: 2
   (i) simple graph, (ii) multigraph, (iii) bipartite graph, (iv) weighted graph
   b) State four-color theorem for planar graph. Prove that if a simple graph is bipartite if and only if it is bicolorable. (A graph is bicolorable if each of its vertices can be assigned one of two different colors such that no two adjacent vertices are assigned with the same color. 3
   c) Draw the following two graphs: $\mathbf{K_5}$, $\mathbf{K_{3,4}}$. 2

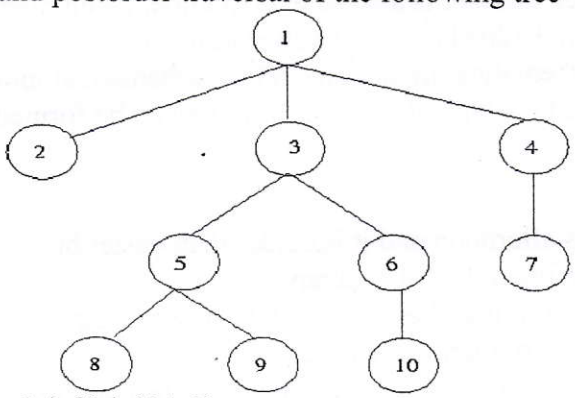
d) Define isomorphism. Are the graphs represented by the following two adjacency matrices isomorphic? 3

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

6. a) Define minimum spanning tree. Construct a minimum spanning tree of the following graph. 4



b) Find the inorder, preorder and postorder traversal of the following tree 3



c) Given an expression: ((5 + z) / -8) * (4 ^ 2) 3

(i) Build a binary tree for the expression

(ii) Write the expression in Postfix Notation.

7. a) State and prove the generalized pigeonhole principle. 3

b) What is the coefficient of x^5y^{10} in the expansion of $(7x - 4y)^{15}$? 2

c) State which of the following graphs are planar. If it is planar then show by drawing it. 2

a. 3-regular graph

b. $\mathbf{K_{3,3}}$

d) Do $\mathbf{K_5}$ and $\mathbf{K_6}$ contain Eulerian circuits? Justify your answer. 2

e) When a graph is called strongly connected? 1