Application of the two-phase simplex method for optimal advertisement decision: A study on an E-commerce company

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Abstract
The optimal advertisement decision aids to get the optimized effective customers within the limited budget. So it plays a significant role for any company. Every e-commerce company has many options for advertisement, but every company has to face difficulties in choosing the best advertisement media for maximization of customers with the restricted expenses. The study is based on a sample e-commerce company which is taken purposively. In this study, I have used the Two-Phase Simplex method which is very much suitable for the study. The study finds that the best media is E-mail, which makes the maximum possible effective customer within the restricted budget.

Key words Two-phase simplex method, Advertising media, E-commerce

Paper type Research paper

1. Introduction

Companies always want to attract more customers within their limited resources. Every company uses different advertising media for reaching its end customers. But reaching the maximum customers using a limited budget is a tough job. For an e-commerce company, it is more than a tough job. Rokomari.com is a Bangladeshi e-commerce company. It uses different advertising media for attracting and acquiring its customers. Influencing media are Facebook post (Boost), Facebook advertisement, Email, SMS, Tech tunes advertisement, and telemarketing. For the last five years, it randomly uses those advertisement media. Using different media different people reaches www.rokomari.com website for purchasing books. Rokomari.com is specialized to sell original books in

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Bangladesh. Fiction, non-fiction and textbooks are available on this site. It has a huge collection of more than 100,000 books. Visitors who visit www.rokomari.com can find their desired books easily from this site. After choosing their books, they can place an order on that site. Rokomari.com arrange those books and make a parcel to send that customer within 2 to 7 days. After getting the parcel, customers have to pay the required amount to the delivery man. This system is called as Cash on Delivery. Those who already know about Rokomari.com are put www.rokomari.com on the search engine address bar and directed to the Rokomoari.com site. But those who don't know about Rokomari.com or at the time of special offer, advertisements are posted on different advertisement media to extract down the interested customer on www.rokomari.com site. To use each advertising media, there is a huge cost involved in it. Price varies from media to media. Thus, it is significant to understand the arena where the number of customers will be maximized with the minimal cost. Rokomari.com collects data from the Google analytics service. Rokomari.com has provided relevant data for this research purpose. Based on the given data and followed by mixed constraints on the set of restrictions, the Two-Phase Simplex Method can easily provide the solution of the maximization of the number of customers with least cost. That will certainly assist the company to maximize the gain. Beside this, this study will help to other policymakers of e-commerce venture in Bangladesh to formulate their action plans in this direction. In the academic arena, this research will be helpful for the students about learning e-commerce, options available for digital advertisement, choosing the best option for the cost minimization and maximization of its effectiveness.

2. Literature review

Different promotions on different media help a company to stimulate the demand for its products. Promotional messages are conveyed to the customers using different advertisement media, i.e., television, magazine, radio, newspapers, billboards, online advertisement, email, social media post. The advertising media can be online or off-line. Any advertisement cuts a huge cost for a company. On the other hand, it is a major concern for the company to minimize the cost and maximize the profit. To get rid of this dilemma many researchers come forward. Keown and Duncan (1979) published a substitute media selection structure and described the outcomes of a descriptive application of IGP (Integer Goal Programming) in their research "Integer Goal Programming in Advertising Media Selection." Paulo Rita, Luiz Moutinho structured a knowledge-based

model 'TOREX' in their research titled "Allocating a Promotion Budget" to assist the national tourism organization. Herein they allocated their promotional budgets by using this model and succeeded to meet their marketing related objectives. Brown and Warshaw (1965) established a general media mix model in "Media Selection by Linear Programming," which assumed linear response and illustrated how the model could be modified to accommodate non-linearity. In "Sequence Matters: A More Effective Way to Use Advertising and Publicity" Loda and Coleman (2005) worked on the sequence of the most effective way of influencing customers in visiting a traveler destination and revealed that the sequence is to be first publicity and then advertising. In "Selection of web sites for online advertising using the AHP" Analytic Hierarchy Process is used in selecting the best website for online promotion and it was provided by Ngai in 2003. This compares and analyses the web sides by adopting a multi-criteria attitude. This method is worked on pairwise evaluation of the several factors that can affect the best website selection. Five factors were used for this comparison: the rate of impression, the periodic monthly cost, the fitness of audience, quality of content and also "look and feel." In "An Application of the Analytic Network Process to the Advertising Media Budget Allocation Decision," Coulter and Sarkis (2009) used ANP (Analytic Network Process; Saaty, 1996) to establish and verify a broad structure of budget allocation as well as media selection. This analytic network process is helpful in solving the multifaceted decision-making problem where it integrates both tangible and intangible measurement into a whole score for classifying the decision alternatives. Lawrence Friedman established game theory and some several mathematical models which describe the allocation of advertising expenditure in his research titled "Game- Theory Models in the allocation of Advertising expenditure". "A Mathematical Modeling in Advertising Budget" studied by Shaikh and Shaikh, it demonstrates a model in the perspective of game theory for two competing cellular companies to assist in their expenses policy to uplift the profits and share in the market. A saddle-point solution is found in the case where a campaign is planned with potentially different marketing areas. In the sense of e-commerce ventures, time demands to work in choosing the best advertisement media selection for cutting down the monetary value and reaches to the maximum effective customers. Undoubtedly, advertisement decision is very important and common phenomenon for any company. So, it is a vital point for any e-commerce company too. Rokomari.com is suffering from this problem. With respect to the advertisement media selection for any e-commerce company in Bangladesh, no research work was done

before. In such a situation this study will be helpful in solving the advertisement media selection problem where there are mixed constraints on the set of restrictions and the Two-Phase Simplex Method can easily find out the optimal solution for such type of problems. This research has given an excellent result on this matter.

3. Objectives of the study
The research objectives are:

1. To identify the best advertising media based on the monthly cost per effective customer
2. To find out the maximum possible effective customer.
3. To find out the optimal advertisement expenditure

4. Methodology
This research study is based on an e-commerce company named Rokomari.com. This e-commerce company uses some online advertising media to acquire maximum effective customers with a limited budget. This study is based on both the primary and the secondary data. The primary data are collected by the regular conversations of the concerned person of the company in the study period. The secondary data are collected from the computer record and also from some manuals. Since the sample company did not preserve the previous data accurately before 2016, that's why the study is conducted based on 2016 data only. The Two-Phase Simplex Method has been used for the data analyzing.

5. Limitations of the study
This research data is collected from only one e-commerce company and this company has been operating here for recent five years. The research data is only based on the year of 2016. Before 2016, this company never used the service of Google Analytics. So it has no specific relevant data before the year 2016 and that's why this research can't show the Trend Line or Time Series Analysis. Hence, Offline or physical advertisement media are excluded from this research work.

6. Theoretical concept of the two-phase simplex method
"Linear Programming is the process of minimizing a linear objective function subject to a finite number of linear equality and inequality constraints" (Karloff, 1991). The mathematical definition of linear programming is simple. It is the analysis of problems in which a linear function of a number of variables is to be maximized (or minimized)

when those variables are subject to a number of restraints in the form of linear inequalities. (Dorfman, Samuelson, & Solow, 1958). Linear programming is the programming of linear function. This function is known as objective function which has to be optimized (maximized or minimized). It consists of a set of constraints while these constraints expressed in linear equality or inequality. Linear programming problem is shortly known as LPP, and it was first introduced by Dantzig in 1947. At that time the United States Air Force Comptroller appointed him as a mathematical advisor for developing a tool for a period organized sending, preparing, and calculated supply program. In spite of the fact that in 1939 the Soviet mathematician and financial specialist Kantorovich figured and tackled this type of problem concerning in organizations, his work stayed obscure until 1959. Subsequently, the origination of the general class of linear programming issues is typically credited to Dantzig. Since the Air Force alludes to its different plans and timetables to be executed as projects, Dantzig's first distributed paper tended to this issue as Programming in a Linear Structure. The expression "linear programming" was developed by the mathematician as well as economist Koopmans in the late spring of 1948 while he met the mathematical advisor Dantzig in California. In the year 1949 Dantzig solved the linear programs by using his invented method the "simplex method." Since that time various people have added to the field of linear programming in a wide range of ways, including hypothetical advancements, computational perspectives, and investigation of new utilization of the subject. The simplex technique for LP appreciates wide acknowledgment given (1) its capacity to demonstrate essential and complex administration choice issues, and (2) its ability for creating arrangements in a sensible measure of time (Bazaraa, Jarvis, & Sherali, 2010). The simplex method, developed by Prof. George B. Dantzig, can be used to solve any L.P problem (for which the solution exists) involving any number of variables and constraints (hundreds or even thousands)(Gupta & Hira, 2014). The simplex method is an iterative technique. The starting of the 1st iteration table is always with an initial feasible solution in this method, and it will be continued until the condition of optimization. The simplex method is based on solutions of simultaneous linear equations. An LP model can be expressed in term of an objective function and a system of linear equations. The constraints form a system of simultaneous linear equations. As a rule, linear programming models have fewer equations than variables. (Stevenson, 1989) Many linear programing problems where slack variables cannot provide such a solution. In these problems, at least one of the constraints is of (≥) or (=) type. In such cases, we introduce

another type of variables called artificial variables. These variables are fictitious and have no physical meaning. They assume the role of slack variables in the first iterations, only to be replaced at a later iteration. Thus, they are merely a device to get the starting basic feasible solution so that simplex algorithm can be applied as usual to get the optimal solution. There are two (closely related) techniques available to solve such problems. They are: 1. The big M– method or M– technique or Method of Penalties with due to A. Charnes and 2. The two-phase method due to Dantzig, Orden, and Wolfe (Gupta & Hira, 2014). If there are mixed constraints on the set of restrictions, then it will be solved by Big-M Method. It is developed by A. Charnes, after his name and it is also known as a Charnes Penalty method. All procedures are same, in the just differ including of basic variables, hence the entering basic variables are surplus variables (≥) and to absorb the non-negativity condition, hence the artificial variables is added to the identity coefficient matrix. These artificial variables are basic variables, which are included also for equal restrictions. The coefficient of artificial variables in the objective function is (–M) for maximization problem and (+M) for minimization problem. It will be solved in one phase. When the LP problem is solved in the computer by using computer programming, then for the computer, it is difficult to manipulate the big M, as a computer can only manipulate a fixed digit, that's why another procedure with the same design in two phases, was developed by G. B. Dantzig, A. Orden and P. Wolfe. The difference in big-M method there was M with artificial variables and in Two Phase Method there will be one instead of M and the procedure is continued by two phases, one phase helps to get the basic feasible solution and the other one helps to get the optimum solution. The following things have been understood while comparing the big-M simplex method and two-phase simplex method.

1. The basic approach to both methods is the same. Both add the artificial variables to get the initial canonical system and then drive them to zero as soon as possible.
2. The sequence of table and the basic change are identical.
3. The numbers of iterations are the same.
4. The big-M method solves the linear program in one phase while the two-phase method solves it in two stages as two linear programs (Sivarethinamohan, 2008).

The Simplex method is a solving way of finding an optimum solution of an initial basic feasible solution. It is a simple iterative procedure. This

optimum basic feasible solution can be obtained by a finite number of iterations. This iteration provides a better feasible solution than the previous one. This procedure continued until an optimum solution is obtained and if it is failed, then, there exists an infeasible or unbounded solution. There are three different types of the simplex method. They are:

i. Simplex Method
ii. Big-M Method
iii. Two Phase Simplex Method

The simplex method is applicable to the problems, including the constraints with (≤) sign. If all the constraints are in the same (≤) sign, then slack variables are included there and by a finite number of iterations we can easily get the optimum solution. To convert the inequality constraint to the equation, we have to add some slack variables which are also known as basic variables (as they contained identity coefficient matrix). The coefficient of these basic variables is zero in the objective function. Others included variables are named as non-basic variable. To get an initial basic feasible solution we have to assume the values of all non-basic variables are zero. The target is to convert the non-basic variables to basic variables which give the optimal solution. To do so, we have to make a table known as simplex table. For this, we have to know some meaning of some symbols like Cj, C_B, B, S, and Zj. Cj is the coefficient of the all variables, including in the objective function. C_B is the coefficient of basic variables in the objective function. B is for the basic variables, S is the initial basic feasible solution. Zj is the summation of the production of each column with the corresponding elements of C_B column. For maximization problem, we have to choose largest positive Cj-Zj and for the minimization case the chosen Cj-Zj will be largest negative value. In the tie case, the most left Cj-Zj will be chosen. Then the selected existing column is called a pivotal column and the value of the initial basic feasible solution will be divided by all the positive value, including the pivotal column. The next step is to find out the minimum ratio, which is got from the quotient. The row containing the minimum ratio is simply known as pivotal row. In the case of a tie in minimum ratio the upper most will be preferable. The element crossing by pivotal row and the pivotal column is the pivotal element which should make to unity. All the elements of the pivotal row, including the solution value will be divided by the value of the pivotal element. And the other elements of the pivotal column will be converted to zero by the row operations. Now the non-basic variable is converted into the basic variables and can enter into

the basic variables list. This procedure will be continued either all Cj-Zj ≤0 (maximization problem) or all Cj-Zj ≥ 0 (minimization problem). In the book of Operations Research by R. Sivarethinamohan, we find that where artificial variables are involved and round off error makes an adverse impact on accuracy, it is better to use the Two-Phase Method. This method solves linear programming problems in two phases.

Phase-I: The first phase helps to get the initial basic feasible solution (IBFS). Indeed, it is very effective to make the shortage of the sum of the artificial variables including in auxiliary objective function. In this phase, a new artificial objective function is introduced in which only the sum of artificial variables are included by $Z = -a_1-a_2-a_3.....-a_n$, all a_i's represent the included artificial variables. And the objective of this function is only minimizing the artificial variables by using the Simplex method. Three cases can arise:

Case-i: Minimum value is zero, then at first all included artificial variables will be zero and the BFS will be derived.

Case-ii: Minimum value is greater than zero and at least one artificial variable is positive, the solution will be infeasible.

Case-iii: All non-negative, i.e., artificial variables are zero then the sum of the variables must be zero, and followed the way for the phase II.

The summarized approach of phase is as below:

Step-1: Check whether the right side of set of restrictions, i.e., the requirement matrix is non-negative or not. If it is negative multiply both sides by -1.

Step-2: To express the set of restrictions in the standard form we should introduce the slack, surplus, and the artificial variables where it is needed in the restrictions.

Step-3: Construct the objective function by introducing the zero with the all variables except the artificial variables and (-1) to all artificial variables.

Step-4: A new objective function (auxiliary objective function) is as Minimize $Z=a_1+ a_2+...+ a_n$, where a_i (i=1, 2,...., m) are the all artificial variables.

Step-5: By using the simplex method, objective function will be minimized and a BFS is obtained. Phase II can be started if there is no artificial variable exists in the optimum solution or if at least one artificial variable contains in the current optimum solution at a zero level. Otherwise, no further improvement will be possible (Infeasible Solution).

Phase II: In this phase, it optimizes the main objective function as usual. The procedure will be followed from the final iteration of Phase I (Basic Feasible Solution).

Step-6: The final iteration table will be the first table of Phase II where the artificial variables vanished or at zero level. Constructing of main objective function is done by including coefficient zero for the artificial variables. The next steps are following the procedure of simplex method and at last table we can get the optimum solution.

In two phase method anyone has to use the simplex method but get the result in the two phases.

7. Model development

7.1 Statement of the advertising media selection problem

An e-commerce company named Rokomari.com wishes to plan its advertising strategy in six different media- Facebook Post (Boost), Facebook Advertisement, Email Marketing, SMS Marketing, and Tech Tunes Advertisement and the Telemarketing. The purpose of advertising is to reach as large a number of effective customers as possible. Following data have been obtained from tracking software:

Particulars	**Media**					
	Face Book Post (Boost)	Face Book Advertisement	E mail Marketing	SMS Marketing	Tech Tunes Advertisement	Telemarketing
Monthly Cost Per Unit (TK)	4000	158000	5000	9900	17000	11200*
Reach To Customers	8120	521481	100000	10000	2871	1400
Monthly Effective Customers (Per Unit)	434	3000	1445	269	98	252

**11200 is the Monthly Salary of the Telemarketer who is working on Rokomari.com and she can call 1400 persons in her 26 working days. And she is the only one person who is dedicated for Telemarketing.*

Rokomari.com wants to spend not more than 400,000 BDT on its Advertising.

Following are the further requirements that must be met:

i. The monthly cost of SMS will be not more than 102,053 BDT
ii. The total budget for both the Facebook post and the Facebook advertisement will be not more than 200,000 BDT
iii. The budget for the Email marketing is not more than 45,000 BDT.
iv. The monthly budget for the telemarketing can be more than 11200 BDT.
v. For the Facebook advertisement the company can spend more than 158,000 BDT.

7.2 Formulation of the linear programming problem

Hence**,** we can assume the monthly output level of each media which are Facebook post, Facebook advertising, E-mail marketing, SMS marketing, Tech Tunes advertisement and the telemarketing are x_1, x_2, x_3, x_4, x_5 and x_6 respectively. Now the advertising Linear Programming problem can be formulated as follows:

$$\text{Max } Z = 434\,x_1 + 3000\,x_2 + 1445\,x_3 + 269\,x_4 + 98\,x_5 + 252\,x_6$$

Subject to,

$$9900\,x_4 \leq 102053$$

$$4000\,x_1 + 158000\,x_2 \leq 200000$$

$$5000\,x_3 \leq 45000$$

$$11200\,x_6 \geq 11200$$

$$4000\,x_1 + 158000\,x_2 + 5000\,x_3 + 9900\,x_4 + 17000\,x_5 + 11200\,x_6 \leq 400000$$

$$158000\,x_2 \geq 158000$$

And $x_1, x_2, x_3, x_4, x_5,$ and $x_6 \geq 0$

8. Solutions to the given effective customers maximization problem

$$\text{Max } Z = 434\,x_1 + 3000\,x_2 + 1445\,x_3 + 269\,x_4 + 98\,x_5 + 252\,x_6 + 0.s_1 + 0.s_2 + 0.s_3 + 0.4 + 0.s_5 + 0.s_6 - 1.a_1 - 1.a_2$$

Subject to, $9900\,x_4 + s_1 = 102053$

$$4000\,x_1 + 158000\,x_2 + s_2 = 200000$$

$$5000\,x_3 + s_3 = 45000$$

$$11200\,x_6 - s_4 + a_1 = 11200$$

$4000\ x_1 + 158000\ x_2 + 5000\ x_3 + 9900\ x_4 + 17000\ x_5 + 11200\ x_6 + s_5 = 400000$

$158000\ x_2 - s_6 + a_2 = 158000$

And $x_1, x_2, x_3, x_4, x_5, x_6, s_1, s_2, s_3, s_4, s_5, s_6, a_1, a_2 \geq 0$

The Initial Basic Feasible Solution,
$s_1 = 102053$, $s_2 = 200000$, $s_3 = 45000$, $a_1 = 11200$, $s_5 = 400000$, $a_2 = 158000$

Result after application of the Two Phase Method, the monthly output level is, $x_1 = 10\frac{1}{2}$, $x_2 = 1$, $x_3 = 9$, $x_4 = 10\frac{3}{10}$, $x_5 = 2\frac{2}{5}$, $x_6 = 1$, Max Z = 23820

That means, the possible maximum customer in each month = 23820

From the above solution, we can come to conclude that, within the limited budget, the monthly cost related to different advertising medium is followed by,

Considering a Facebook post,
The company can spend monthly $= 10\frac{1}{2} \times 4000$ BDT
$= 42{,}000$ BDT

Considering Facebook Advertisement,
The company can spend monthly $= 1 \times 158{,}000$ BDT
$= 158{,}000$ BDT

Considering Email Marketing,
The company can spend monthly $= 5{,}000 \times 9$ BDT
$= 45{,}000$BDT

Considering SMS Marketing,
The company can spend monthly $= 10\frac{3}{10} \times 9{,}900$ BDT
$= 101{,}970$ BDT

Considering Tech Tunes Advertisement,
The company can spend monthly $= 2\frac{2}{5} \times 17{,}000$ BDT
$= 40{,}800$ BDT

Considering Telemarketing,
The company can spend monthly $= 1 \times 11{,}200$ BDT
$= 11{,}200$ BDT

So, the total monthly cost

$= (42{,}000 + 158{,}000 + 45{,}000 + 101{,}970 + 40{,}800 + 11{,}200)$ BDT
$= 398{,}970$ (within its monthly budget) BDT
And the maximum monthly effective customer is = 23820 Persons.

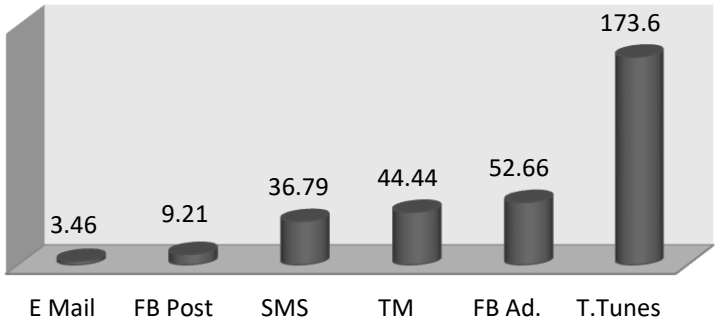
Findings and Interpretations of Results

Media	Monthly Advertising units	Monthly Advertising Expenditure (BDT)	Monthly Maximum Effective Customers	Monthly Cost Per Effective Customer (BDT)
Facebook post (Boost)	$10\frac{1}{2}$	42,000	4557	9.21
Facebook Advertisement	1	158,000	3000	52.66
E Mail	9	45,000	13005	3.46
SMS	$10\frac{3}{10}$	101,970	2771	36.79
Tech Tunes Advertisement	$2\frac{2}{5}$	40,800	235	173.61
Telemarketing	1*	11,200	252	44.44
Total		398,970	23820	

**11, 200 refers to 1 Unit*

Priority list based on Monthly Cost per Effective Customer (BDT)

Media	Monthly Cost Per Effective Customer (BDT)
E Mail	3.46
Facebook post (Boost)	9.21
SMS	36.79
Telemarketing	44.44
Facebook Advertisement	52.66
Tech Tunes Advertisement	173.6



Rokmari.com should provide great effort on email marketing. To make a customer from its visitor Rokomari.com has to spend only 3.46 BDT, if

it's done advertisement through email marketing. It can also put emphasize on Facebook Post (Boost) as it has to spend only 9.21 BDT for bringing one customer. In the next priority SMS will come forward as it is less cheap than the rest three and the per customer acquisition cost is 44.44 BDT. It can also keep the Facebook advertisement media due to its customer acquisition and branding purposes. It highly helps to promote the brand value of that company. Rokomari.com should keep an eye for Tech Tune media, because it is cutting a huge cost in comparison, which is 173.6 BDT for per customer acquisition. It may be better for Rokomari.com to stop spending in the Tech Tune and investing that amount in E Mail Marketing and Facebook Post (Boost). This decision will help Rokomari.com not only to get customer in lower cost but also for creating brand image for its visitors and customers or, Rokomari.com may appoint more three Telemarketers for telemarketing instead of expending Tech Tunes Advertisement. If Rokomari.com wants to keep the entire Media active, then it has to distribute its 400,000 BDT budget as follows:

Media	Monthly Advertising Expenditure (BDT)
E-mail	45,000
Facebook post (Boost)	42,000
SMS	101,970
Telemarketing	11,200
Facebook Advertisement	158,000
Tech Tunes Advertisement	40,800

The total cost will be 398,970 BDT and this distributed expenditure will help to get maximum 23820 (app) customers. The rest of the amount, which is 1030 BDT can be used for either E-mail Marketing or Facebook Post (boost). Both Media will help to get valuable customers and company brand promotion.

9. Conclusion

Many companies are spending their valuable money on advertising media without proper calculation, thus it can give the best output. The Two Phase method is surely able to help on effective customer maximization within a limited budget for advertising. Not only e-commerce but also other types of companies can use this method to get solution for their effective customer maximization within their limited budget for advertising.

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Appendices

Table-1		Cj	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	Min Ratio
C_B	B	S	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6	a_1	a_2	
0	s_1	102053	0	0	0	9900	0	0	1	0	0	0	0	0	0	0	-
0	s_2	200000	4000	158000	0	0	0	0	0	1	0	0	0	0	0	0	1.26
0	s_3	45000	0	0	5000	0	0	0	0	0	1	0	0	0	0	0	-
0	s_5	400000	4000	158000	5000	9900	17000	11200	0	0	0	0	1	0	0	0	2.53
-1	a_2	158000	0	158000	0	0	0	0	0	0	0	0	0	-1	0	1	1
-1	a_1	11200	0	0	0	0	0	11200	0	0	0	-1	0	0	1	0	-
	Zj	-169200	0	-158000	0	0	0	-11200	0	0	0	1	0	1	-1	-1	-
		Cj-Zj	0	158000	0	0	0	11200	0	0	0	-1	0	-1	0	0	

Phase-I:

Table-2		Cj	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	Min Ratio
C_B	B	S	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6	a_1	a_2	
0	s_1	102053	0	0	0	9900	0	0	1	0	0	0	0	0	0	0	-
0	s_2	42000	4000	0	0	0	0	0	0	1	0	0	0	0	0	0	-
0	s_3	45000	0	0	5000	0	0	0	0	0	1	0	0	0	0	0	-
0	s_5	242000	4000	0	5000	9900	17000	11200	0	0	0	0	1	0	0	0	21.60
0	x_2	1	0	1	0	0	0	0	0	0	0	0	0	-1/158000	0	1/158000	-
-1	a_1	11200	0	0	0	0	0	11200	0	0	0	-1	0	0	1	0	1
	Zj	-11200	0	0	0	0	0	-11200	0	0	0	1	0	0	-1	0	-
		Cj-Zj	0	0	0	0	0	11200	0	0	0	-1	0	0	0	0	-

Table-3		Cj	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1
C_B	B	S	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6	a_1	a_2
0	s_1	102053	0	0	0	9900	0	0	1	0	0	0	0	0	0	0
0	s_2	42000	4000	0	0	0	0	0	0	1	0	0	0	0	0	0
0	s_3	45000	0	0	5000	0	0	0	0	0	1	0	0	0	0	0
0	s_5	230800	4000	0	5000	9900	17000	0	0	0	0	0	1	0	0	0
0	x_2	1	0	1	0	0	0	0	0	0	0	0	0	-1/158000	0	1/15800
0	x_6	1	0	0	0	0	0	1	0	0	0	-1/11200	0	0	1/11200	0
	Zj	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		Cj-Zj	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1

As all Cj-Zj ≤0, so the basic feasible solution exists here.

Phase-II

Table-4		Cj	434	3000	1445	269	98	252	0	0	0	0	0	0	Min Ratio
C_B	B	S	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6	
0	s_1	102053	0	0	0	9900	0	0	1	0	0	0	0	0	-
0	s_2	42000	4000	0	0	0	0	0	0	1	0	0	0	0	-
0	s_3	45000	0	0	5000	0	0	0	0	0	1	0	0	0	9
0	s_5	230800	4000	0	5000	9900	17000	0	0	0	0	0	1	0	46.16
3000	x_2	1	0	1	0	0	0	0	0	0	0	0	0	-1/158000	-
252	x_6	1	0	0	0	0	0	1	0	0	0	-1/11200	0	0	-
	Zj	3252	0	3000	0	0	0	252	0	0	0	-63/2800	0	-3/158	
		Cj-Zj	434	0	1445	269	98	0	0	0	0	63/2800	0	3/158	

Table-5		Cj	434	3000	1445	269	98	252	0	0	0	0	0	0	Min Ratio
C_B	B	S	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6	
0	s_1	102053	0	0	0	9900	0	0	1	0	0	0	0	0	-
0	s_2	42000	4000	0	0	0	0	0	0	1	0	0	0	0	10.5
1445	x_3	9	0	0	1	0	0	0	0	0	1/5000	0	0	0	-
0	s_5	185000	4000	0	0	9900	17000	0	0	0	-1	0	1	0	46.25
3000	x_2	1	0	1	0	0	0	0	0	0	0	0	0	-1/158000	-
252	x_6	1	0	0	0	0	0	1	0	0	0	-1/11200	0	0	-
	Zj	16257	0	3000	1445	0	0	252	0	0	1445/5000	-63/2800	0	-3/158	
		Cj-Zj	434	0	0	269	98	0	0	0	-1445/5000	63/2800	0	3/158	

Table-6		Cj	434	3000	1445	269	98	252	0	0	0	0	0	0	Min Ratio
C_B	B	S	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6	
0	s_1	102053	0	0	0	9900	0	0	1	0	0	0	0	0	10.31
434	x_1	$10\frac{1}{2}$	1	0	0	0	0	0	0	1/4000	0	0	0	0	-
1445	x_3	9	0	0	1	0	0	0	0	0	1/5000	0	0	0	-
0	s_5	143000	0	0	0	9900	17000	0	0	-1	-1	0	1	0	14.44
3000	x_2	1	0	1	0	0	0	0	0	0	0	0	0	-1/158000	-
252	x_6	1	0	0	0	0	0	1	0	0	0	-1/11200	0	0	-
	Zj	20814	434	3000	1445	0	0	252	0	434/4000	1445/5000	-63/2800	0	-3/158	
		Cj-Zj	0	0	0	269	98	0	0	0	-1445/5000	63/2800	0	3/158	

Table-7		Cj	434	3000	1445	269	98	252	0	0	0	0	0	0	Min Ratio
C_B	B	S	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6	
269	x_4	$10\frac{3}{10}$	0	0	0	1	0	0	1/9900	0	0	0	0	0	-
434	x_1	$10\frac{1}{2}$	1	0	0	0	0	0	0	1/4000	0	0	0	0	-
1445	x_3	9	0	0	1	0	0	0	0	0	1/5000	0	0	0	-
0	s_5	40947	0	0	0	0	17000	0	-1	-1	-1	0	1	0	2.48
3000	x_2	1	0	1	0	0	0	0	0	0	0	0	0	-1/158000	-
252	x_6	1	0	0	0	0	0	1	0	0	0	-1/11200	0	0	-
	Zj	$23587\frac{39}{100}$	434	3000	1445	269	0	252	269/9900	434/4000	1445/5000	-63/2800	0	-3/158	
		Cj-Zj	0	0	0	0	98	0	-269/9900	-434/4000	-1445/5000	63/2800	0	3/158	

Table-8		Cj	434	3000	1445	269	98	252	0	0	0	0	0	0
C_B	B	S	x_1	x_2	x_3	x_4	x_5	x_6	s_1	s_2	s_3	s_4	s_5	s_6
269	x_4	$10\frac{3}{10}$	0	0	0	1	0	0	1/9900	0	0	0	0	0
434	x_1	$10\frac{1}{2}$	1	0	0	0	0	0	0	1/4000	0	0	0	0
1445	x_3	9	0	0	1	0	0	0	0	0	1/5000	0	0	0
98	x_5	$2\frac{2}{5}$	0	0	0	0	1	0	-1/17000	-1/1700	-1/1700	0	1/17000	0
3000	x_2	1	0	1	0	0	0	0	0	0	0	0	0	-1/158000
252	x_6	1	0	0	0	0	0	1	0	0	0	-1/11200	0	0
	Zj	23820	434	3000	1445	269	98	252	21/500000	27/250	36/125	-63/2800	49/8500	-3/158
		Cj-Zj	0	0	0	0	0	0	-21/500000	-27/250	-36/125	63/2800	-49/8500	3/158

Since all Cj-Zj ≤ 0 and the optimality conditions are satisfied, hence the optimum solution is obtained for the given advertising linear programming problem.