আন্তর্জাতিক ইসলামী বিশ্ববিদ্যালয় চট্টগ্রাম

الجامعة الإسلامية العالمية شيتاغونغ

International Islamic University Chittagong

Department of Civil Engineering, FSE

Mid Term

SEMESTER- Autumn- 2022 SESSION
MATH 2405- Linear Algebra, Matrices and Vector Analysis

Programme	**: Bachelor of Civil Engineering**	**Level of Study**	**: UG 2**
Time	**: 2.30~4.00 PM**	**Date**	**: 24th September 2022 Saturday**
Duration	**: One hour thirty minutes**		
Course Code	**: MATH--2405**	**Section(s)**	**: AM**
Course Title	**: Linear Algebra, Matrices and Vector Analysis**		

This Question Paper Consists of **Two (2)** Printed Pages (Including Cover Page) with **3 (Three)** Questions.

Course Learning Outcome(CLO)	
CLO1	Demonstrate the basic idea of vector spaces, subspaces, Linear dependence and independence of vectors, Linear mappings, Inner product spaces and be able to find the eigenvalues and eigenvectors of a square matrix using the characteristic polynomial and will know how to diagonalize a matrix.
CLO2	Get the basic understanding about scalar and vectors, dot Product, cross product derivative of vectors, vector integration.

Bloom's Levels of the Questions					
Letter Symbol	R	A	ANA	E	C
Meaning	**Remember (5)**	**Apply(5)**	**Analysis(5)**	**Evaluation(10)**	**Create (5)**

INSTRUCTION(S) TO CANDIDATES

DO NOT OPEN UNTIL YOU ARE ASKED TO DO SO

- This is a Closed Book, Closed Notes Examination.
- Do not open until You Are Asked to Do So.
- Attempt All Questions. All Questions Carry Equal Marks.
- Use of Programmable or Graphical Calculator is Strictly Not Allowed.
- Mobile Phones and Other Electronic Devices are Prohibited in the Exam Hall.
- **Work Can Be Done Using Pencil Or Pen, But The Final Answer Must Be Written In Pen**

Note: Any form of cheating or attempt to cheat is a serious offence, which may lead to dismissal

QUESTION 1 (10 marks)		
a)	Define Linear combination of vectors. Express (if possible) the matrix $E=\begin{pmatrix}3 & 1\\ 1 & -1\end{pmatrix}$ as a linear combination of the matrix $A=\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}$, $B=\begin{pmatrix}0 & 0\\ 1 & 1\end{pmatrix}$ and $C=\begin{pmatrix}0 & -2\\ 0 & 1\end{pmatrix}$.	4 marks
b)	Define linear dependence and linear independence of vectors. Determine whether or not the vectors in $\mathbf{R}^3$ are linearly dependent or independent: (1,-2,1),(2,1,-1),(7,-4,1) and (2,-3,7),(0,0,0),(3,-1,-4)	3 marks
c)	Define Vector sub space. Let v=$\mathbf{R}^3$.Show that W is a subspace of V where W={a,b,c : a+b+c=0} that is, W consists of those vectors each of the property that the sum of its components is zero.	3 marks

QUESTION 2 (10 marks)		
a)	Define sum and direct sum of two subspaces. Determine the value of K such that the system has (i) a unique solution (ii) no solution (iii) more than one solution $x+y-z=1,\ 2x+3y+\ Kz=3,\ \ x+Ky+3z=2$	3 marks
b)	Define linear mapping. Let V be the vector space of n-square matrices over k. Let M be an arbitrary matrix in V. Let $T: V \to V$ be defined by $T(A)=AM+MA$, Where $A \in V$. Show that T is linear. But $F: R^2 \to R$ defined by $F(x,y)=xy$ is not linear.	4 marks
c)	Define a basis and dimension of a vector space. $F: R^3 \to R^3$ be the linear mapping defined by $T(x,y,z)=(x+2y-z, y+z, x-y+2z)$ Find a basis and the dimension of the image $U\ of\ T$	3 marks

QUESTION 3 (10 marks)		
a)	Use the Gram Schmidt orthogonalization process to construct the basis $\{u_1,u_2,u_3\}$into an orthogonal basis where u_1=(1,1,1), u_2=(0,0,1) and u_3=(1,0,1)	6 marks
b)	Define Inner Product Spaces and write its axioms. Verify that the following is an inner product in R^2 $<u,v>=x_1y_1-x_1y_2-x_2y_1+3x_2y_2$, where $u=(x_1,x_2), v=(y_1,y_2)$	4 marks

End of the Mid Term Questions