**International Islamic University Chittagong**

**Department of Computer Science & Engineering**

*B. Sc. in CSE Semester Final Examination, Spring-2018*

**Course Code: CSE-1223 Course Title:** Discrete Mathematics

Total marks: ***50*** Time: ***2*** hours ***30*** minutes

[Answer any ***two*** from **Group-A** and any ***three*** from **Group-B** of the following questions. Figures in the right-hand margin indicate full marks. Use Separate answer script for Group-A and Group-B.]

## Group-A

1. a) What is a composite number? Show that if n is a composite integer, then n has a prime divisor less than or equal to $\sqrt{n}$. 3
   b) Write the Euclidean algorithm to find the Greatest Common Divisor. 2
   c) Express gcd(252, 198) as a linear combination of 252 and 198. Show the steps. 3
   d) What is *linear congruence*? Solve the congruence $4x \equiv 5 \pmod 7$. 2

2. a) Define with example: 2
   i) Direct proof and ii) Proof by contradiction
   b) Give a direct proof of the theorem "If **n** is an odd number, then $\mathbf{n^2}$ is odd." 2
   c) Write the *principle of mathematical induction*. Use mathematical induction to show that 3 divides $n^3 + 2n$ whenever n is a positive integer. 3
   d) Prove that every amount of postage of 12 Taka or more can be formed using just 4 Taka and 5 Taka stamps. 3

3. a) What do you mean by *rules of inference*? Construct an argument using rules of inference to show that the hypotheses "Randy works hard", "If Randy works hard, then he is a dull boy" and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job". 2
   b) Write short notes on: 2
   i) Theorem ii) Conjecture.
   c) Define *recursion*. Suppose $f$ is defined recursively by the following form: 3
   $f(0) = f(1) = 1$.
   $f(n+1) = f(n) - f(n-1)$ for $n \geq 1$.
   Find $f(3)$ and $f(7)$.
   d) Solve the congruence equation $48x \equiv 284 \pmod{356}$. 3

## Group-B

4. a) Suppose that there are 9 faculty members in the Mathematics department and 11 in the Computer Science department. How many ways are there to select a committee to develop a Discrete Mathematics course at a school if the committee is to consist of 3 faculty members from the Mathematics department and 4 from the Computer Science department? 2
   b) What do you mean by the *Inclusion- Exclusion principle*? How many bit strings of length 10 either begins with three 0s or ends with two 0s. 3
   c) Suppose an ice-cream shop has three different types of ice-cream. How many different ways can seven ice-creams be chosen. 2

d) State and prove the *generalized pigeonhole principle*. Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4. 3

5. a) State the *Handshaking theorem* of graph. Verify this theorem for the Fig-5(a). 2

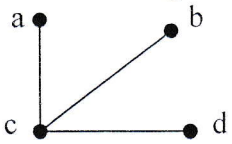


Fig – 5(a)

b) Define with example – 3
i) Complete graph ii) Bipartite graph iii) Connected graph

c) What do you mean by *adjacency matrix*? Draw the directed graph by the following adjacency matrix. 2

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

d) What do you mean by *isomorphism of graphs*? Determine whether the given pair of graphs in Fig-5(d) is isomorphic- 3


Fig-5(d)

6. a) What is *m-ary* tree? How many edges does a *full binary tree* with 100 internal vertices have? 2

b) Consider the algebraic expression $\mathbf{E = (2x + y)(5a - b)^3}$. 4
a) Draw the tree **T** which corresponds to the expression **E**.
b) Find the *preorder* and *postorder* of T.

c) What is *spanning tree* and *minimum spanning tree*? Construct a minimum spanning tree of the following graph in Fig-6(c). 4

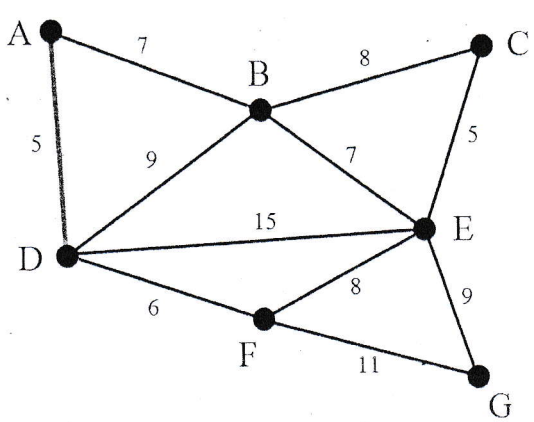


Fig-6(c)

7. a) A multiple-choice test contains 10 questions. There are four possible answers for each question. 2

i) How many ways can a student answer the questions on the test if every question is answered?

ii) How many ways can a student answer the questions on the test if the student can leave answers blank?

b) Define an *Euler circuit* and an *Euler path* in an undirected graph. Determine whether the following graph in Fig-7(b) has an Euler circuit or path. Construct such a circuit or path if it exists. 3

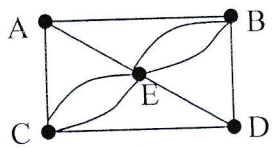


Fig 7(b)

c) What do you mean by *planar graph*? Determine whether the graph in Fig-7(c) is planar. If so, draw it so that no edges cross. 3

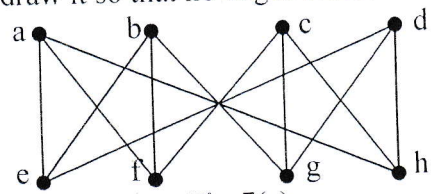


Fig-7(c)

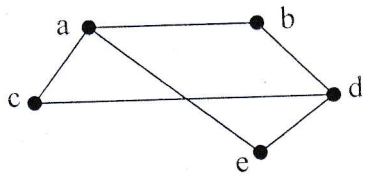


Fig- 7(d)

d) State *Four color theorem*. Find the *chromatic number* of the graph shown in Fig-7(d). 2