# International Islamic University Chittagong

**Department of Computer Science and Engineering**

*B. Sc. in CSE Final Examination, Autumn 2021*

**Course Code: CSE 4747 Course Title: Mathematical Analysis for Computer Science**

*Total marks: **50*** *Time: **2 hours 30 minutes***

The figures in the right hand margin indicate full marks.
Course Outcomes and Bloom's Taxonomy Levels are mentioned in additional Column

## Group A

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|---|---|---|---|---|
| 1a) | In a best-of-three tournament, the probability of the IIUC football team to win the first match is 50%. In subsequent games, their probability of winning is determined by the outcome of the previous game. If the IIUC team lost the previous game, then they work hard on tactics and win the current game with a probability of 60%. If they win the previous game, then they keep less focus on the game and win the current game with a probability of only 30%.<br>i) What is the probability that the first two matches decide the tournament champion?<br>ii) What is the probability that the IIUC team wins the tournament, given that they win the first game? | CO3 | E | 6 |
| 1b) | Suppose that we flip three fair, mutually-independent coins and consider the following three events:<br>• A1 is the event that coin 1 matches coin 2.<br>• A2 is the event that coin 2 matches coin 3.<br>• A3 is the event that coin 3 matches coin 1.<br>Now, illustrate that how pairwise independence does not necessarily imply the mutual independence. | CO2 | An | 4 |
| | Or, | | | |
| 1b) | Find the probability that in a family of 4 children there will be<br>(a) at least 1 boy and<br>(b) at least 1 boy and 1 girl. Assume that the probability of a male birth is 0.5 | CO2 | An | 4 |
| 2a) | A computer program crashes at the end of each hour of use with probability $p$, if it has not crashed already. Now, formulate a procedure to estimate the expected time until the program crashes? | CO2 | An | 4 |
| 2b) | At an unknown railway station, a person arrived at a four-way intersection point. Now, he wants to find out the exit gate, but there is no proper signboard available in this station. Therefore, initially, he has to choose one of four directions. If he goes North, then he will find the exit gate after one minute of travelling. If he goes East, he will wander around the station for three minutes and will then return to his initial position. If he goes South, he will wander around the station for ten minutes and will then return to his initial position. If he goes West, then he will find the exit gate after five minutes of travelling. Assuming that the person is at all times equally likely to choose any of the four directions. Now, estimate the expected number of minutes that the person will be trapped in the station. | CO3 | E | 6 |
| | Or, | | | |
| 2b) | Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.8; if it rained today but not yesterday, then it will rain tomorrow with probability 0.6; if it rained yesterday but not today, then it will rain tomorrow with probability 0.3; if it has not rained in the past two days, then it will rain tomorrow with probability 0.1. | CO3 | E | 6 |

(a) Represent the scenario with a four-state Markov chain transition probability matrix.

(b) Given that it rained on Monday and Tuesday, what is the probability that it will rain on Thursday?

## Group B

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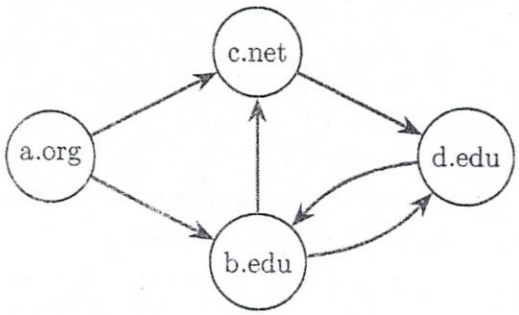
|---|---|---|---|---|
| 3a) | What is Markov property. | CO1 | R | 2 |
| 3b) | A wireless channel is either bad (due to noise and interference) or good. A particular channel changes its states from slot to slot as follows: a good channel continues to be good in the next slot with probability 0.7, and turns bad with probability 0.3; a bad channel becomes good in the following slot with probability 0.6 and remains bad with probability 0.4.<br>What is the probability that a channel will be found good in the eleventh slot if it is observed to be bad in the seventh slot?<br>What are the probabilities that, in an arbitrary slot in the long run, the channel will be found in good or bad states? | CO3 | E | 8 |
| 4a) | State the gambler's ruin problem. Formulate a mathematical model to estimate the probability of avoiding ruins. | CO1 | R | 3 |
| 4b) | Formulate a procedure for robust calculation of the ranks of the pages in a Web-graph based on a random walk on that graph.<br>Illustrate how it is meant to work with the following Webgraph where (hyper)links are represented by directed edges. | CO3 | U | 3<br>4 |




Make reasonable assumptions as necessary.

| | | | | |
|---|---|---|---|---|
| 5a) | Explain the Simpson's paradox. | CO1 | R | 3 |
| 5b) | Following slotted Aloha protocol, $n$ contending nodes attempt to transmit via a shared channel to an access point. The probability that a given node transmits in a particular slot is $p$. The nodes act independently of each other, and the events across slots are also independent. Any slot that sees more than one (concurrent) transmission is wasted due to collision, and slots that don't have any transmission from any of the $n$ nodes are also wasted idly. A single transmission from one and only one of the nodes results in a successful transmission. With this setup, find the followings:<br>i) If there is a collision in a slot, what is the expected number of nodes involved in the collision?<br>ii) What is the probability that a given slot sees at least one transmission? | CO3 | Ap | 7 |

Or,

| | | | | |
|---|---|---|---|---|
| 5a) | What is queueing model? Describe some of the characteristics of it. | CO1 | R | 3 |
| 5b) | A tool crib has exponential interarrival and service times, and it serves a very large group of mechanics. The mean time between arrivals is 4 times. It takes 3 minutes on the average for a tool-crib attendant to service a mechanic. The attendant is pain \$10 per hour and the mechanic is pain \$15 per hour. Would it be advisable to have a second tool-crib attendant? | CO3 | Ap | 7 |