

International Islamic University Chittagong (IIUC)

Department of Electronic and Telecommunications Engineering

Final Examination

Program: B.Sc. (Engg.)
Course Code: Math-3503

Semester: Spring 2018
Course Title: Math V
(Coordinate geometry and higher trigonometry)
Time: 2 hours 30 minutes

Full Marks: 50

Group- A

[Answer *any two* sets of the following questions]

1.	(a)	Show that the two lines $x=ay+b=cz+d$ and $x=\alpha y+\beta=\gamma z+\delta$ will be coplanar if $(\gamma-c)(\alpha\beta-b\alpha)=(\alpha-a)(c\delta-\gamma d)$	04
	(b)	Find the condition that the lines $x-az-b=0=y-cz-d$ and $x-mz-n=0=y-pz-w$ be perpendicular	03
	(c)	Find the equation of the plane which passes through the point (1,2,3) and parallel to each $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x+5}{-1} = \frac{y-3}{2} = \frac{z-1}{1}$	03
2.	(a)	Find the equation of the plane which contains the line $x = \frac{y-3}{2} = \frac{z-5}{3}$ which is perpendicular to the plane $2x + 7y - 3z = 1$	05
	(b)	Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$; $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. Find also the equations and the points in which it meets the given lines.	05
3.	(a)	If the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the co-ordinate axes in the points A, B, C and O be the origin. Find the equation of the sphere OABC	05
	(b)	If any tangent plane to the sphere $x^2 + y^2 + z^2 = r^2$ makes intercepts a, b, c on the co-ordinate axes, prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{r^2}$	05
Group- B			
[Answer <i>any three</i> sets of the following questions]			
4.	(a)	Prove that for integral values of n, positive or negative, $\cos n\theta + i \sin n\theta$ is equal to $(\cos \theta + i \sin \theta)^n$	04
	(b)	Find all values of $(\sqrt{3} + i)^{1/5}$	03
	(c)	if $x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$; prove that $x_1 x_2 x_3 \dots$ to infinity = -1	03
5.	(a)	Using De Morgan's theorem solve the equation $(x + 1)^5 + (x - 1)^5 = 0$	04
	(b)	Prove that $\frac{\sin^3 \theta}{3!} = \frac{\theta^3}{3!} - (1 + 3^2) \frac{\theta^5}{5!} + \dots$	03
	(c)	$x = \frac{2}{1!} - \frac{4}{3!} + \frac{6}{5!} - \frac{8}{7!}$ and $y = 1 + \frac{2}{1!} - \frac{2^3}{3!} + \frac{2^4}{5!} - \dots$ Show that $x^2 = y$	03

6.	(a)	Solve: $\sec^{-1} \frac{x}{a} + \sec^{-1} \frac{x}{b} = \sec^{-1} b - \sec^{-1} a$	03
	(b)	Show that $\cosh^2 \theta - \sinh^2 \theta = 1$	03
	(c)	If $\cos^{-1}(u + iv) = \alpha + i\beta$ then prove that $\cos^2 \alpha$ and $\cosh^2 \beta$ are the roots of $x^2 - (1 + u^2 + v^2)x + u^2 = 0$	04
7.	(a)	If $\tan(\alpha + i\beta) = x + iy$ prove that i. $x^2 + y^2 + 2x \cot 2\alpha = 1$ ii. $x^2 + y^2 - 2y \cot 2\beta + 1 = 0$	3+3
	(b)	If $\sin^{-1}(u + iv) = \alpha + i\beta$; prove that $\sin^2 \alpha$ and $\cosh^2 \beta$ are the roots of $x^2 - (1 + u^2 + v^2)x + u^2 = 0$	04