International Islamic University Chittagong
Department of Electrical and Electronic Engineering

Final Examination Autumn-2018
Course Code: **MATH-3505**
Time: **2 hours 30 minutes**

Program: B.Sc. Engg. (EEE)
Course Title: **Mathematics-V**
Full Marks: **50**

Figures in the right margin indicate full marks. Use separate script for each part.

Part - A

[Answer any ***two*** sets from the following questions]

1(a). Construct the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 2 \end{bmatrix}$ to echelon form and find its rank. **2**

1(b). Define Hermitian matrix with example. If $A = \begin{pmatrix} 1-3i & 5+8i \\ -3 & 3-7i \\ -6-i & 5i \end{pmatrix}$, then find the Hermitian matrix A^H. **3**

1(c). Define Inverse matrix. Solve the following linear system by using matrix inversion method: **5**

$$x + 2y + 3z = 1$$
$$x + 3y + 6z = 3$$
$$2x + 6y + 13z = 4$$

2(a). State Cayley Hamilton's theorem. Verify Cayley Hamilton's theorem for the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. **4**

2(b). Determine the Characteristic polynomial, Eigen values and Eigen vector for the following matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$. **6**

3(a). Reduce $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ to normal form and find its rank. **3**

3(b). If $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$, then prove that its Diagonalization is $D = P^{-1} A\, P$, where P is its Eigen vector matrix. **5**

3(c). Prove that $A = \begin{pmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{pmatrix}$ is orthogonal. **2**

Part - B

[Answer any ***three*** sets from the following questions]

4(a). Determine λ and μ by using vectors, such that the points $(-1, 3, 2), (-4, 2, -2)$ and $(5, \lambda, \mu)$ lie on a straight line. **3**

4(b). A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time. **4**

- **i.** Determine its velocity and acceleration at any time.
- **ii.** Find the magnitudes of the velocity and acceleration at $t = 0$.

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Part - B

[Answer any ***three*** sets from the following questions]

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4(b). A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time. **4**

- **i.** Determine its velocity and acceleration at any time.
- **ii.** Find the magnitudes of the velocity and acceleration at $t = 0$.

4(c). Show that the vectors $2\,\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}, \boldsymbol{i} - 3\boldsymbol{j} - 5\boldsymbol{k}$ and $3\boldsymbol{i} - 4\boldsymbol{j} - 4\boldsymbol{k}$ form the sides of a right angled-triangle. 3

5(a). If $\boldsymbol{A} = (3x^2 + 6y)\boldsymbol{i} - 14yz\boldsymbol{j} + 20xz^2\boldsymbol{k}$, evaluate $\int_C \boldsymbol{A}.d\boldsymbol{r}$ from (0, 0, 0) to (1, 1, 1) along the following paths C. 5

i. $x = t, y = t^2, z = t^3$.

ii. The straight line from (0, 0, 0) to (1, 0, 0), then to (1, 1, 0), and then to (1, 1, 1).

iii. The straight line joining (0, 0, 0) and (1, 1, 1).

5(b). Explain the physical significance of Gradient, Divergence and Curl. Determine the constant a so that the vector $\mathbf{V} = (x + 3y)\mathbf{i} + (y - 2z)\mathbf{j} + (x + az)\mathbf{k}$ be solenoidal . 2

5(c). Compute $\int_C \boldsymbol{F}.d\boldsymbol{r}$, where $\boldsymbol{F} = \frac{iy - jx}{x^2+y^2}$ and $\boldsymbol{C}$ i[illegible] $x^2 + y^2 = 1$ trav[illegible] counter clockwise. [illegible]

6(a). State Green's theorem. Verify Green's theorem in the plane for $\oint_C (xy + y^2)\,dx + x^2dy$ where C i[illegible] region bounded by $y = x$ and [illegible] 4

6(b). By using Stok[illegible] s is the surf[illegible] $z = 1 - x^2 - y^2,\ z \geq 0, \boldsymbol{n}$ is the unit normal to the

[illegible] cylindrical coordinate system is orthogonal. 3

7(a). A fluid motion is given by $\mathbf{V} = (x + 2y + a[illegible])\mathbf{i} + (bx - 3y - z)\mathbf{j} + (4x + c[illegible])$ For what value of a, b, c, will the motion [illegible] 3

7(b). Apply the Di[illegible]ce theorem to e[illegible] is the surface of the cylinder $x^2 + y^2 = a^2$ bounded by the planes z = 0, z = b and where $\boldsymbol{u} = ix - jy + kz$.

7(c). If $A = (x^2y)i - 2yzj + 2yzk$, evaluate curl curl A. 3