International Islamic University Chittagong
Department of Electrical and Electronic Engineering

Final Examination Spring-2018 | Program: B.Sc. Engg. (EEE)
Course Code: Math-2404 | Course Title: Math-IV
Time: 2 hours 30 minutes | Full Marks: 50

Part A

[Answer any ***two*** questions from the followings; figures in the right margin indicate full marks.]

1(a). If $f(z)$ has a simple pole at $z = a$, then prove that $res\, f(a) = \lim_{z\to a}(z-a)f(z)$ **3**

1(b). Find the residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z = 1$ **4**

1(c). Evaluate $\int_0^{\pi} \frac{d\theta}{3+2cos\theta}$ by applying residue theorem in the complex plane. **3**

2(a). Prove that $\mathcal{L}\{t^n\} = \frac{\Gamma(n+1)}{s^{n+1}}$ **2**

2(b). Using the Laplace transforms, find the solution of the initial value problem : $Y'' + 9Y = cos\ 2t, Y(0) = 1, Y(\frac{\pi}{2}) = -1$ **4**

2(c). If $L\frac{d^2i}{dt^2} - R\frac{di}{dt} + \frac{1}{C}i = v(t)$ is the equation of the following RLC circuit then, prove that its current $i(t) = -8e^{2t} + 17t\, e^{2t} + 8\cos 2t$, when $L = 1H, R = 4\Omega, c = \frac{1}{4}\ \mu F$ and $i(0) = 0, i'(o) = 1$ **4**

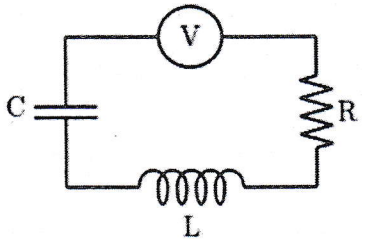

Figure-1

3(a). Evaluate the following integral using residue theorem $\int_c \frac{z^2}{(z-1)^2(z+2)}dz$: where $|z| = 3$ **3**

3(b). Form the first principle, find the Laplace transform of $(1 + \cos 2t)$ **3**

3(c). Prove that $\mathcal{L}[f'(t)] = s\mathcal{L}[f(t)] - f(0)$ **4**

Part B

[Answer any ***three*** questions from the followings; figures in the right margin indicate full marks.]

4(a). State and Prove Convolution theorem 4

4(b). Express the following function in terms of units step function and find its Laplace transform $f(t) = \begin{cases} 8 & when\ t < 2 \\ 6 & when\ t > 2 \end{cases}$ 3

4(c). If $f(t) = \left(\frac{2t}{3}\right), where\ 0 \leq t \leq 3$ is a periodic function find it's Laplace transform. 3

5(a). Give the definition of Fourier series and define the a_0, a_n, b_n 2

5(b). Find the Fourier series for the function $f(x) = x, when\ 0 \leq x \leq 2\pi$ and sketch the function from $x = -6\pi$ to 6π 4

5(c). Find the Fourier series of the function $f(x) = \begin{cases} -1 & for \quad -\pi < x < -\frac{\pi}{2} \\ 0 & for \quad -\frac{\pi}{2} < x < \frac{\pi}{2} \\ +1 & for \quad \frac{\pi}{2} < x < \pi \end{cases}$ 4

6(a). Give the definition of Fourier integral transform. 1

6(b). Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & when\ |x| \leq 1 \\ 0 & when\ |x| > 1 \end{cases}$ 5

6(c). Define Z transform, then find the Z transformation: 4

$$f(k) = \begin{cases} 0 & when\ k < 0 \\ 1 & when\ k \geq 0 \end{cases}$$

7(a). What is the Impulse function, if $\delta(t-a) = \begin{cases} \frac{1}{\epsilon} & , \ a < t < a + \epsilon \\ 0 & otherwise \end{cases}$ 3

is a unit impulse function then prove that its Laplace transform is $f(\eta)$

7(b). Find the Fourier series to represent $f(x) = \pi - x, when\ 0 < x < 2\pi$ 4

7(c). Find the Fourier *sine* transforms of $f(x) = e^{-ax}$ 3