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# Crashing in the Optimal Project Management: Cut Search Method Vs. Linear Programming Method

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***Abstract:*** *The paper compares Cut Search Method and Linear Programming Method to the project time-cost trade-off analysis as a practical application and a conceptual development. Both the solution procedures are presented in this paper to determine the minimum resource utilization requirement to reduce the total project completion time from a normal to a crash condition. The study would help management in selecting the appropriate method for getting optimal crashing decision. The study finds Linear Programming method is better than Cut Search method. Linear Programming approach is also very much suitable to a broad range of practical project management problems, which provide some flexibility. Finally, the study suggests for using Linear Programming Method to the crashing project duration of urgent projects in the pipeline of ADP and private sector projects as well.*

***Key Words:*** *crashing project Management, Cut Search method, Linear Programming*

## 1. Introduction:

The project management time-cost tradeoff analysis addresses the issue as to "which activity or activities in a project should be allocated additional resources in order to reduce the total project duration by a required amount of time at a minimal level of additional resource expenditure (Philips, 1996)?" In any project type situation the main concern is time duration and resource level for completing the project in an expected time involved or for completing the project in less time than the expected (normal) time. This reduced completion time interval can only be achieved by incurring additional costs due to the allocation of supplemental resources. A project is defined as a set of activities that

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must be completed in a prearranged sequence. The Project Management Institute defines project management as " ...the art of directing and coordinating human and material resources throughout the life of a project by using modern management techniques to achieve predetermined objectives of scope, cost, time, quality and participant satisfaction" (**Cleland,** 1994). Over the years, the project management process is getting more complicated and is difficult to complete it without the aid of systems directed at efficient management of time and cost. There is a continuous need for the project management to expand and improve its capabilities and its scope of operations to meet changing and ever growing demands for its success. However, project managers are continuously facing the situation in which they must take a decision to complete the project within the stipulated time and cost with the required standard of quality, safety and other specifications. Minimizing project completion time and cost continues to be a universally sought and highly desirable goal. Therefore, the main objective of the project management is to minimize the total project time subject to the resource constraints.

Project management has evolved as a new field with the development of two popular analytical quantitative techniques that help managers plan, schedule, monitor, and control large and complex projects. These two techniques are the PERT and CPM. PERT is an acronym for the Programme Evaluation and Review Technique, developed in 1957-58 in response to the needs of the Polaris Fleet Ballistic Missile project of the US Government. CPM is an acronym for the Critical Path Method. Akin to PERT, it was developed independently in 1956-57 by the Du Pont Company in the US to solve scheduling problems in industrial settings. Although the two methods were developed independently they are striking similar and in many respects they are indistinguishable. Thus, PERT and CPM comprise one technique and historically CPM was based on deterministic times while PERT was based on probabilistic times. Consequently both techniques will be referred to as project scheduling techniques (**Anderson, Sweeney** and **Williams**, 2000).

PERT and CPM provide some valuable information to the management. The CPM is a very effective tool for obtaining useful information like the total completion time of a project, the critical activities, the non-critical activities, and the total, free and independent slack of various non-critical activities. A significant aspect of the CPM network analysis lies in its capacity to evaluate alternate ways to expedite some (or possibly all) of the activities indicated in a network and then analyze their cost involvements. This analysis is referred to as the time-cost trade-off analysis. The time-cost trade-off problem is based on the conception that the duration of some of the activities of a project can be cut down, if some additional resources – men, materials and / or equipments are employed on them. Such deliberate reduction of activity times by putting extra effort is called crashing. Naturally, crashing costs are more than normal costs and managers are usually interested in speeding up a project at the least additional cost. This approach of project crashing is applicable with critical path method, where both the time to complete each activity and the cost of doing so are known with certainty. CPM uses two sets of time and cost estimates for each activity i.e. normal time and normal cost; and crash time and crash cost. The crash time is the shortest possible activity time and crash cost is the money of completing the activity on a crash or deadline basis.

## 2. Statement of the Problem:

Optimal project management is very essential for any developing country like Bangladesh as because of all developing countries suffer from paucity of investible funds. Bangladesh is dependent to a large extent on the outside world for many of its regular needs and development aids. 48.40% of the country's annual development expenditures are made up of foreign assistance (GOB, 2006). Nearly half of the total aid in Bangladesh is channeled as project aid (**Chadha**, 1989). Unfortunately project management in Bangladesh is not up to the mark. Sometimes the projects are not completed but are dropped from ADP (Annual Development Programme). Non-implementation or delayed implementation of project is a common phenomenon in Bangladesh. In any consideration time is money and thus any delay in project schedule entails an increase of project costs. Not only that some unusual factors, i.e. change of value of money due to inflation and devaluation etc. affect project costs also. Project actual cost more than planned is seen almost everywhere but the situation is more serious with respect to project implementation in Bangladesh. **Hoque** (1987) and **Chadha** (1989) showed the actual scenario in managing projects in Bangladesh. They provided a clear picture of the real time overrun and cost overrun in implementing projects in Bangladesh (Annexure 1 and 2). **Chadha** (1989) studied 29 projects and found that the average cost of delay per project was US $117 million. **Hoque** (1987) identified that the project implementation and control are poor in Bangladesh due to unusual delay at the various stages of its implementation. Poor project management enormously increased the completion periods and in turn, total costs of the projects leading to over-capitalization of the projects. Over capitalization and heavy use of long-term loans in financing total investments of the projects produce adverse impact on the interest of the different stakeholders of the enterprises. This serious problem can be resolved by reducing project duration with some additional resources and thus all projects will be completed within the stipulated time and therefore, we can save the huge amount of resources.

For doing this, there are two methods, namely, Cut Search Method and Linear Programming Method for crashing schedule a project. From these two, which one will be more appropriate this is the vital question to a project manager. This study attempts to identify the better one for crashing in project management. Thus, this study would be helpful and relevant in the perspective of delayed implementation of projects in Bangladesh.

## 3. Objectives of the Study:

The main objective of the paper is to compare, focus and provide a framework for reducing project time at the least cost by using Cut Search Method and Linear Programming Method. The study also attempts to examine both the methods of crashing in identifying a suitable method that satisfies necessary requirements of a project manager and provides the most accurate method.

## 4. Data and Methodology of the Study:

The study is mainly of desk nature and based on the secondary data. The main sources of data are different published books, journals, bulletins, magazines etc. related to crashing in optimal project management. In this study, for more clear understanding, a hypothetical project data have been used to formulate and to get an optimal solution to minimize the project crashing cost with both the methods Cut Search Method and LP Method. We have solved the formulated project management problem by Cut Search Method manually and LP method by Management Scientist (MS) software.

## 5. Theoretical Concepts of Crashing in Project Management:

To crash the project duration with minimum cost, the two available methods of crashing are discussed below:

### Cut Search Method:

This approach is considered as manual approach. In this approach, it is required to determine incremental cost for each activity. The incremental cost for an activity can be determined by using the following equation:

$$\text{Incremental Cost} = \frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal Time} - \text{Crash Time}}.$$

The first step is to identify and crash the critical activity that has the minimum incremental cost of crashing. In the event of multiple critical paths, an activity from each such path is chosen. Of the various combinations available, the one with the least cost is selected. In particular, it may be economical to consider joint critical ones – activities that are common to two or more critical paths. In each case, the crashing is done for one unit time (by a day, week, month etc.)

In the second step, the network is revised by adjusting the time and cost of the crashed activity. The critical path(s) is identified again, and we revert to the first step. This process is continued till no more crashing of the project is feasible. The feasible crash time of a project is determined by the length of the critical path using crash activity times.

Now the optimal duration of the project can be determined. It would be the time duration corresponding to which the total cost – direct cost plus indirect cost -- is the minimum.

**Linear Programming Method**:

Linear Programming is another approach to finding the best project-crashing schedule. Linear programming solution gives the minimum amount of additional resources that are needed to spend to complete the project by the stipulated time.

Let $x_i$ = time represented by node *i*.

$y_j$ = amount of crash time used on activity *j*.

$k_j$ = Crash cost per unit of time for activity *j*.

The objective is to minimize the cost of crashing the total project and thus the LP objective function will be:

$$\text{Minimize } Z = \sum_j k_j y_j$$

Subject to,

(i) Project completion constraint: This constraint specifies that the last event must take place before the project deadline time. If a project must be completed by time s, then

$x_n \leq s$ (where *n* is the highest numbered node).

(ii) Crash time constraints: Constraints are required to ensure that each activity is not crashed more than its maximum allowable crash time. The maximum for each Y- variable is the difference between the normal time and the crash time. For each activity, the amount of crash time cannot exceed the amount available; thus for each activity *j*:

i.e. $y_j \le M_j$ [where *M* is maximum allowable crash time]

(iii) Constraints describing the network: The final set of constraints describes the structure of the network. Every activity will have one constraint for each of its predecessors. Each node represents the maximum of the earliest finish times of the activities with arrows into this node; thus for activity, *j*, between nodes *k* and *i*:

$x_i \ge x_k$ + (Time to complete activity *j*)

i.e. $x_i \ge x_k + (\tau_j - y_j)$; (There is one constraint for each activity).

(iv) Non-negativity constraints: All variables are non-negative:

$x_i, y_j \ge 0$ for all *i* and *j*.

### 6. A Hypothetical Project Management Problem:

For better understanding of the concept, formulation and solution procedure, we consider a hypothetical project management problem. Let us assume that a project is the combination of four activities, which are interrelated in a logical sequence in the sense that the starting of some activities are dependent on the completion of some other activities. The immediate predecessors, normal times, normal costs, crash times and crash costs of these four activities are given in Table –1. The project network diagram according to the given information is drawn and is shown in Figure-1. Table 2 shows the Earliest Start Time (EST), Earliest Finish Time (EFT), Latest Start Time (LST), Latest Finish Time (LFT) and slack times for all of the activities. Slack time is the length of time an activity can be delayed without affecting the project completion time. A sequence of consecutive activities with zero slack time forms a critical path. From Table-2, it is seen that the critical activities are B, C and D and the critical path is B – C – D and thus the project completion time is 24 days. If any one of the critical activities is delayed for any reason, the entire project must be delayed. The total project cost for a normal completion time is fixed at $810 (Table –1); we can minimize the total project cost (normal cost plus crashing cost) through crashing.

**Table 1: A Project with Hypothetical Data**

| Activities | Immediate Predecessor | Normal Time (Days) | Crash Time (Days) | Normal Cost ($) | Crash Cost ($) |
|---|---|---|---|---|---|
| A | -- | 4 | 3 | 80 | 105 |
| B | -- | 6 | 4 | 180 | 250 |
| C | B | 8 | 5 | 200 | 320 |
| D | A, C | 10 | 6 | 350 | 530 |

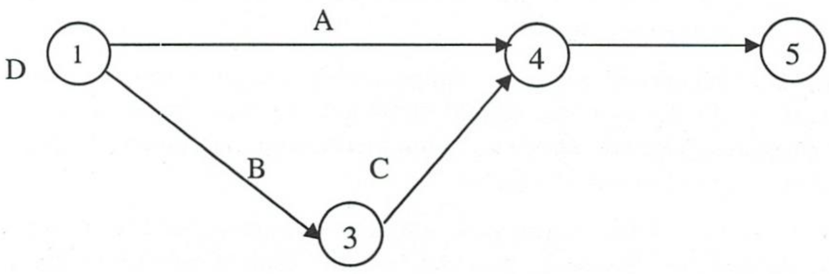


Figure1: The Network diagram of the project.

**Table 2: Activity Schedule for the Project**

| Activities | EST | EFT | LST | LFT | Slack =LST-EST | Critical Path |
|---|---|---|---|---|---|---|
| A | 0 | 4 | 10 | 14 | 10 | No |
| B | 0 | 6 | 0 | 6 | 0 | Yes |
| C | 6 | 14 | 6 | 14 | 0 | Yes |
| D | 14 | 24 | 14 | 24 | 0 | Yes |

Source: Calculated by the authors.

**Table 3: Activity Incremental Cost (Time-Cost Slope) Per unit of time for Crashing**

| Activities | Activity time can be Crashed | Additional Cost for Crashing | Crash Cost Per Unit of Time |
|---|---|---|---|
| A | 1 | 25 | 25 |
| B | 2 | 70 | 35 |
| C | 3 | 120 | 40 |
| D | 4 | 180 | 45 |

Source: Calculated by the authors.

Let the management wants to complete this project within 18 days. Therefore, the management should know how to crash the activities of the project so that the total cost is minimum. The incremental costs of each activity are shown in Table-3.

The crash time of the critical path activities (all crash plan) is 15 days. This implies that this model provides feasible result if only set deadline of the project is greater than or equal to 15 days. Management's desire to complete the project within 18 days lies in between the all crash and all normal plans (durations).

## 7. Findings of Crashing Schedule of the Project:

### 7.1. Cut Search Method for Crashing the Project:

In order to reduce the project duration by Cut Search Method, we have to crash the activities on the critical path only. From Table – 3, it is seen that the activity B has the lowest crash cost per day, so it becomes the first candidate for crash and thus activity B has crashed by 2 days. Since the crash limit for critical activity B is reached, we consider critical activity C with the next lowest crash cost per day. After crashing activity C for 3 days then we consider critical activity D with the next lowest cost per day. Although we can reduce D by 4 days, but it is only required to reduce it by 1 day to reach our project completion goal of 18 days. In different crash schedule, critical activities, cost slopes against critical activities, respective decision, project duration, critical path and extra cost involvement etc. are shown in Table –4.

**Table 4: Calculation of crashing cost by using Cut Search Method.**

| Crash schedule | Critical activity | Cost per week | Decision | Project completion time | Critical path | Additional costs |
|---|---|---|---|---|---|---|
| 1 | B<br>C<br>D | 35<br>40<br>45 | Crash activity *B* by 2 days. | (24 – 2) = 22 days. | B-C-D | 70 |
| 2 | C<br>D | 40<br>45 | Crash activity *C* by 3 days. | (22 – 3) = 19 days. | B-C-D | 120 |
| 3 | D | 45 | Crash activity *D* by 1 day. | (19 – 1) = 18 days. | B-C-D | 45 |
| Total crashing cost | | | | | | 235 |

Source: Calculated by the authors.

### 7. 2. Linear Programming Model Development for Crashing the Project:

To develop a linear programming model for the sample project problem, we consider the following components:

Objective function: Minimize Z = Total additional cost.

Subject to,

i. The project completion time is 18 days.

ii. No more crash time is used than is available for each activity.

iii. Times at nodes$\geq$ (time at preceding node) + (time to do activity including crash time).

iv. Non-negativity restrictions.

Now, let $x_i$ = the earliest finish time for activity *i*; *i* = A, B, C, D, E.

$y_i$ = the amount of time activity *i* is crashed; *i* = A, B, C, D, E.

Thus the model can be presented as,

Minimize $Z = 25y_A + 35y_B + 40y_C + 45y_D$

Subject to ,

$$x_D \leq 18 \text{ .......... .......... } (i)$$

$$\left.\begin{array}{l} y_A \leq 1 \\ y_B \leq 2 \\ y_C \leq 3 \\ y_D \leq 4 \end{array}\right\} \text{.............................} (ii)$$

$$\left.\begin{array}{lll} x_A \geq 0 + (4 - y_A) & \text{i.e.} & x_A + y_A \geq 4 \\ x_B \geq 0 + (6 - y_B) & \text{i.e} & x_B + y_B \geq 6 \\ x_C \geq x_B + (8 - y_C) & \text{i.e} & x_C + y_C - x_B \geq 8 \\ x_D + y_D - x_A \geq 10 & \text{i.e} & x_D + y_D - x_A \geq 10 \\ x_D + y_D - x_C \geq 10 & \text{i.e} & x_D + y_D - x_C \geq 10 \end{array}\right\} \text{....................} (iii)$$

$$\left.\begin{array}{ll} x_K \geq 0; & K = A, B, C, D \\ y_K \geq 0; & K = A, B, C, D \end{array}\right\} \text{.........................} (iv)$$

If the management sets the deadline of the project completion time any one of the 15 to 24 days then the model would be developed accordingly i.e. just by changing equation (i).

## Interpretation of the LP Model Results:

For solving such a linear programming model sophisticated computer software can be used. Microsoft Excel Solver can also be used for the same purpose. To determine the optimal crashing for each of the activities, we solve the above project-management-linear-programming-problem of 8 variables, 10 constraints by using Management Scientist Software which is developed by Anderson et al (2000). The solution of the model is given in Table – 5 and Table – 6. Table – 5 shows the optimal solution of crashing activity $B$ by 2 days, $C$ by 3 days and activity $D$ by 1 day with a total crashing cost of \$235.

**Table 5: Optimal Solution: Values and Reduced Cost of the Variables**

(Objective function value = $235.00)

| Variable | Value | Reduced Costs |
|---|---|---|
| $x_A$ | 9.00 | 0.00 |
| $x_B$ | 4.00 | 0.00 |
| $x_C$ | 9.00 | 0.00 |
| $x_D$ | 18.00 | 0.00 |
| $y_A$ | 0.00 | 25.00 |
| $y_B$ | 2.00 | 0.00 |
| $y_C$ | 3.00 | 0.00 |
| $y_D$ | 1.00 | 0.00 |

Source: Computer printout.

**Table 6: Optimal Solution: Slack / Surplus and dual prices of the Resources**

| Constraint | Slack / Surplus | Dual Prices |
|---|---|---|
| 1 | 0.00 | 45.00 |
| 2 | 1.00 | 0.00 |
| 3 | 0.00 | 10.00 |
| 4 | 0.00 | 5.00 |
| 5 | 3.00 | 0.00 |
| 6 | 5.00 | 0.00 |
| 7 | 0.00 | -45.00 |
| 8 | 0.00 | -45.00 |
| 9 | 0.00 | 0.00 |
| 10 | 0.00 | -45.00 |

Source: Computer printout.

Sensitivity analysis i.e. the range of feasibility (right hand side values ranges) and the range of optimality (objective function co-efficient ranges) of the project management problem are given in Table – 7 and

Table - 8. The current crash cost per day for activity B is $35. Table – 7 shows that the range of optimality for activity B is $0.00 to $45. This implies that if the crash cost for activity B varies from this range, then the optimal solution would be remain optimum. But if the cost for activity B exceeds $45 then the solution will no longer be optimal and then the new linear programming model should be developed to search for an optimal solution. Similarly, other ranges for activity C and D can be interpreted and are given in the same table.

**Table 7: Sensitivity Analysis: Ranges of Optimality**

(Objective Function Co-efficient Ranges)

| Variable | Lower Limit | Current Value | Upper Limit |
|---|---|---|---|
| $x_A$ | -5.00 | 0.00 | 0.00 |
| $x_B$ | -10.00 | 0.00 | No Upper Limit |
| $x_C$ | -5.00 | 0.00 | No Upper Limit |
| $x_D$ | No Lower Limit | 0.00 | 45.00 |
| $y_A$ | 0.00 | 25.00 | No Upper Limit |
| $y_B$ | No Lower Limit | 35.00 | 45.00 |
| $y_C$ | No Lower Limit | 40.00 | 45.00 |
| $y_D$ | 40.00 | 45.00 | No Upper Limit |

Source: Computer printout.

Table-8 shows the ranges of feasibility (resource range) for 10 constraint equations. In the first constraint, the current project completion time is 18 days. The range of feasibility for this resource is 15 days to 19 days. This indicates that if the project completion time were within the range then the optimal solution would remain be optimal. But if the project completion time falls from 15 days or exceeds 19 days then the solution will no longer be optimal and then the new linear programming model should be developed to search for an optimal solution. Similarly, the remaining other resource ranges can be interpreted and are given in the same table.

**Table 8: Sensitivity Analysis: Ranges of Feasibility**

(Right Hand Side Values Ranges)

| Constraint | Lower Limit | Current Value | Upper Limit |
|---|---|---|---|
| 1 | 15.00 | 18.00 | 19.00 |
| 2 | 0.00 | 1.00 | No Upper Limit |
| 3 | 0.00 | 2.00 | 3.00 |
| 4 | 0.00 | 3.00 | 4.00 |
| 5 | 1.00 | 4.00 | No Upper Limit |
| 6 | No Lower Limit | 4.00 | 9.00 |
| 7 | 5.00 | 6.00 | 9.00 |
| 8 | 7.00 | 8.00 | 11.00 |
| 9 | No Lower Limit | 10.00 | 15.00 |
| 10 | 9.00 | 10.00 | 13.00 |

Source: Computer printout.

## 8. Conclusion:

The paper presents the comparison of both cut search method and Linear Programming method to the project time-cost tradeoff analysis as a practical application and a conceptual development. Both the solution procedures are presented in this paper determines the minimum resource utilization requirement to reduce the total project completion time from a normal to a crash condition. In crashing project duration, Cut search method is a manual method or trial and error method and it is a time consuming erroneous process. For small and simple networks, this trial and error approach can be used to make crashing decisions; in large and complex networks, however, this approach is difficult and impractical, and a mathematical procedure is required to determine the optimal crashing decisions. Therefore, more sophisticated mathematical technique i.e. linear programming model can be employed in this direction. It is clear that LP method would help management to reach the optimality where sensitivity analysis would provide some flexibility in the model. This approach is applicable to a broad range of practical project management problems. Therefore, LP method is more suitable, appropriate, accurate, and flexible method in crashing of the project management than Cut Search Method.

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## Appendix

### **Annexure 1(a):** Project Implementation Records (1987-88)

| Duration of Delay | Number of Projects |
|---|---|
| 1-5 years | 96 |
| 6-10 years | 38 |
| 11-15 years | 12 |
| 16 years and above | 5 |

Source: Chadha (1989)

### **Annexure 1(b):** Project Implementation Records (1987-88)

| Sample Project | Budgeted period | Actual period | Duration of Delay |
|---|---|---|---|
| 1 | 2 years | 3 years | 1 year |
| 2 | 2 years 6 months | 4 years 6 months | 2 year |
| 3 | 3 years 9 months | 5 years 6 months | 1year 9 months |
| 4 | 4years | 5 years 6 months | 1year 6 months |
| 5 | 4 years 3 months | 5 years 3 months | 1 year |

Source: Hoque (1987)

**Annexure 2(a):** Project Cost Overrun (1987-88)

| Cost Overrun | Number of Projects |
|---|---|
| 100 – 200% | 57 |
| 200 – 300% | 7 |
| 300 – 400% | 6 |
| 400 – 500% | 1 |
| 500 – 600% | 2 |
| 600 – 700% | 3 |
| Over 700% | 6 |

Source: Chadha (1989)

**Annexure 2(b):** Project Cost Overrun (1987-88) (in Million TK.)

| Sample Project | Original Budgeted Cost | Revised Budgeted Cost | Actual Cost |
|---|---|---|---|
| 1 | 142.1 | 149.9 | 221.9 |
| 2 | 73.7 | 102.3 | 156.7 |
| 3 | 25.5 | 55.2 | 55.7 |
| 4 | 219.5 | 371.1 | 474.0 |
| 5 | 269.5 | 312.7 | 312.7 |

Source: Hoque (1987)