International Islamic University Chittagong
Department of Electrical and Electronic Engineering

Final Examination Spring-2018 | Program: B.Sc. Engg. (EEE)
Course Code: **Math-3505** | Course Title: **Mathematics V**
Time: **2 hours 30 minutes** | Full Marks: **50**

Use separate script for each part. Figures in the right margin indicate full marks.

Part - A

[Answer any ***two*** questions from the followings.]

1(a). Show that the matrix $A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$ is an Idempotent Matrix. **2**

1(b). Determine the Rank of the following matrix $A = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 3 & 8 \\ 0 & 0 & -2 \end{pmatrix}$. **3**

1(c). Define inverse matrix. Find the inverse of the matrix $A = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ by using row elementary matrix operation. **5**

2(a). If $A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{pmatrix}$, then find its characteristic equation (Polynomial) and Eigen values (characteristic roots). **2**

2(b). Using Cayley Hamilton theorem, find the A^{-1} of the matrix $A = \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix}$. **2**

2(c). If a square matrix $A = \begin{pmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$, find the modal matrix P and the resulting diagonal matrix D of A. **6**

3(a). Show that $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ is an involutory matrix. **2**

3(b). Find the solution of the following system of linear equations **4**

$$x + y + z = 3, \qquad x + 2y + 3z = 4, \qquad x + 4y + 9z = 6$$

by using matrix analysis (inverse).

3(c). If $A = \begin{pmatrix} -5 & -2 \\ 4 & 1 \end{pmatrix}$, then find it's Eigen values and Eigen vectors. **4**

Part - B

[Answer any ***three*** questions from the followings.]

4(a). Forces of magnitudes 5 and 3 units acting in the directions $6\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ and $3\boldsymbol{i} - 2\boldsymbol{j} + 6\boldsymbol{k}$, respectively, act on a particle which is displaced from the point $(2, 2, -1)$ to $(4, 3, 1)$. Find the work done by the forces. **3**

4(b). Prove that $\boldsymbol{a}.(\boldsymbol{b}\times\boldsymbol{c}) = \boldsymbol{b}.(\boldsymbol{c}\times\boldsymbol{a}) = \boldsymbol{c}.(\boldsymbol{a}\times\boldsymbol{b})$ 3

4(c). Prove that $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) = (\boldsymbol{a}.\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}.\boldsymbol{b})\boldsymbol{c}$ 4

5(a). Evaluate $\boldsymbol{\nabla\phi} = \log(x^2+y^2+z^2)$. 3

5(b). Find the directional derivative of the scalar function $f(x,y,z) = x^2+xy+z^2$ at the point $A(1,-1,-1)$ in the direction of the line AB where B has co-ordinates (3, 2, 1). 3

5(c). Show that the line integral $\int_{(1,2)}^{(3,4)}(xy^2+y^3)dx+(x^2y+3xy^2)dy$ is independent of the joining points $(1,2)$ and $(3,4)$. Hence, evaluate the integral. 4

6(a). State Stoke's theorem. 1

6(b). Prove the Green's theorem. 5

6(c). Use the Divergence theorem to evaluate $\iint_s (xdydz+ydzdx+zdxdy)$, where S is the portion of the plane $x+2y+3z=6$ which lies in the first octant. 4

7(a). Find the unit vector normal to surface $\phi(x^2+3y^2+2z-6)$ at $P(2,0,1)$. 3

7(b). Find the volume of the parallelepiped if $\boldsymbol{a} = -3\boldsymbol{i}+7\boldsymbol{j}+5\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{i}+7\boldsymbol{J}-3\boldsymbol{k}$, and $\boldsymbol{c} = 7\boldsymbol{i}-5\boldsymbol{j}-3\boldsymbol{k}$ are the three co-terminus edges of the parallelepiped. 3

7(c). Using Green's theorem, evaluate $\int_c (x^2+xy)dx+(x^2+y^2)dy$, where C is the square formed by the lines $y = \pm1,\ x = \pm1$. 4