

**International Islamic University Chittagong**  
**Department of Electrical and Electronic Engineering**

**Final Examination Autumn-2018**

Course Code: MATH-2309

Time: 2 hours 30 minutes

Program: B.Sc. Engg. (EEE)

Course Title: Mathematics-III

Full Marks: 50

**Part A**

[Answer any two questions from the followings; figures in the right margin indicate full marks.]

- 1(a). Find the Characteristic polynomial, Eigen values and Eigen vector for the following matrix 03  
 $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$
- 1(b). Let  $A = \begin{pmatrix} 1-3i & 5+8i \\ -3 & 3-7i \\ -6-i & 5i \end{pmatrix}$  then find  $A^H$ . 03
- 1(c). If  $A = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$ , then find  $A^3$  and  $A^{-1}$ . 04
- 2(a). State Cayley Hamilton's theorem. Verify Cayley Hamilton's theorem for the matrix 06  
 $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ . Also find  $A^{-1}$
- 2(b). Define Eigen value and Eigen vectors. Find the Eigen value and corresponding Eigen 04  
vectors of the matrix  $A = \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}$
- 3(a). What is canonical form of a matrix? Construct the matrix  $A = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 2 \end{pmatrix}$  to 03  
canonical form and find its rank.
- 3(b). Define diagonalization of matrices. Diagonalize the matrix  $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$  05
- 3(c). What is orthogonal matrix? If A is orthogonal matrix then show that  $A^{-1}$  is also 02  
orthogonal.

**Part B**

[Answer any three questions from the followings; figures in the right margin indicate full marks.]

- 4(a). Find the area of the triangle having vertices at  $P(1,3,2)$ ,  $Q(2,-1,1)$  and  $R(-1,2,3)$  03
- 4(b). If  $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ ,  $\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  and  $\vec{C} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ , then Show that 03

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \text{ and then Prove that } \vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$$

- 4(c). Prove that the law of *sine's angle* for a plane triangle by using vector. 04
- 5(a). Illustrate the physical significance of Gradient, Divergence and Curl. 04  
 Prove that  $\nabla^2\left(\frac{1}{r}\right) = 0$
- 5(b). If  $F = (3xy^2)\mathbf{i} - y^2\mathbf{j}$ , evaluate  $\int_C F \cdot d\mathbf{r}$  where C is the curve in the xy plane  $y = 8x^2$ , from (0,0) to (1,2) 03
- 5(c). A fluid motion is given by  $V = (x+y+az)\mathbf{i} + (bx+3y-z)\mathbf{j} + (3x+cy+z)\mathbf{k}$  for what value of a,b,c the motion is irrotational? 03
- 6(a). State Green's theorem. Verify Green's theorem in the plane for  $\oint_C (xy + y^2) dx + x^2 dy$  where C is the closed curve bounded by  $y = x$  and  $y = 8x^2$ . 05
- 6(b). Prove that a rectangular coordinate system is orthogonal. 03
- 6(c). Apply Gauss's divergence theorem to evaluate  $\iiint_S \mathbf{u} \cdot \mathbf{n} ds$ , where S is the surface of the cylinder  $x^2 + y^2 = a^2$  bounded by the planes  $z=0, z=h$  and where  $\mathbf{u} = x\mathbf{i} - y\mathbf{j} - kz\mathbf{k}$ . 02
- 7(a). What is solenoidal? Justify the vector  $V = (x+3y)\mathbf{i} + (y-2z)\mathbf{j} + (x-2z)\mathbf{k}$  is solenoidal. 03
- 7(b). If  $A =$  04  
 C. (i) The straight line from (0,0,0) to (1,0,0), then to (1,1,0), and (1,1,1), (ii) straight line joining (0,0,0) and (1,1,1).
- 7(c). If  $\phi = 2xyz^2, F = xy\mathbf{i} - z\mathbf{j} + x^2\mathbf{k}$  and C is the curve  $x=t^2, y=2t, z=t^3$  from  $t=0$  to  $t=1$ , evaluate line integrals  $\int_C F \times d\mathbf{r}$