International Islamic University Chittagong
Department of Electrical and Electronic Engineering

Final Examination Autumn-2018
Course Code: Math-1202
Time: 2 hours 30 minutes

Program: B.Sc. Engg. (EEE)
Course Title: Math-II
Full Marks: 50

Part A

[Answer any ***two*** questions from the followings; figures in the right margin indicate full marks.]

1(a). **Determine** the angle between the line $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{6}$ and the plane $3x + y + z - 1 = 0$. 5

1(b). Find the length and the equations of the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{1}$ and $\frac{x-5}{3} = \frac{y-5}{2} = \frac{z+1}{-5}$. 5

2(a). **Define** great circle. **Find** the equation of the sphere through the points $(0, 0, 0)$, $(0, 1, -1)$, $(-1, 2, 0)$ and $(1, 2, 3)$. Locate its center and **find** the radius. 5

2(b). Show that the condition for spheres 5

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

and $x^2 + y^2 + z^2 + 2u'x + 2v'y + 2w'z + d' = 0$

to cut orthogonally is $2uu' + 2vv' + 2ww' = d + d'$.

3(a). Find the equation to the cone whose vertex is the origin and which passes through the curve of the intersection of the plane $lx + my + nz = p$ and the surface $ax^2 + by^2 + cz^2 = 1$ 3

3(b). **Find** the equations of the spheres passing through the circle $x^2 + y^2 + z^2 - 6x - 2z + 5 = 0$, $y = 0$ and touching the plane $3y + 4z + 5 = 0$. 5

3(c). Find the equation of the right circular cylinder of radius 2, whose axis is the line $\frac{1}{2}(x - 1) = \frac{y}{3} = z - 3$ 2

Part B

[Answer any ***three*** questions from the followings; figures in the right margin indicate full marks.]

4(a). If n is an integer, **prove** that 4

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

4(b). Solve the equation $x^5 - 1 = 0$ using De Moivre's theorem. 3

4(c). If n is a positive integer, show that $\left(\sqrt{3} + i\right)^n + \left(\sqrt{3} - i\right)^n = 2^{n+1}\cos\frac{n\pi}{6}$. 3

5(a). If p, q are two roots of $t^2 - 2t + 2 = 0$, show that 4

$$\frac{(x+p)^n - (x+q)^n}{p-q} = \sin n\theta \operatorname{cosec}^n\theta, \text{ where } \tan\theta = \frac{1}{1+x}.$$

5(b). If $x = \log\tan\left(\frac{\pi}{4}+\frac{y}{2}\right)$, **prove** that $y = -i\log\tan\left(\frac{\pi}{4}+\frac{ix}{2}\right)$. 3

5(c). If $\tan\log(x+iy) = a+ib$, where $a^2+b^2 \neq 1$, **prove** that 3

$$\tan\log(x^2+y^2) = \frac{2a}{1-a^2-b^2}.$$

6(a). **Express** $\sin\left(\frac{\pi}{4}+x\right)\cos x$ as a power series in x. 3

6(b). $\frac{}{1!}-\frac{}{3!}+\frac{5}{5!}-\cdots\cdots$ and $y = 2-\frac{4}{3!}+\frac{6}{5!}-\cdots\cdots$, **prove** that 4

$x = y^2$.

6(c). If $\frac{}{\theta} = \frac{2165}{2166}$, **show** that the value of θ is approximately 3^0. 3

7(a). **Evaluate** the sum of the following series: 5

$$\tan^{-1}\frac{x}{1+1\cdot 2x^2}+\tan^{-1}\frac{x}{1+2\cdot 3x^2}+\tan^{-1}\frac{x}{1+3\cdot 4x^2}+\cdots\cdots n-th\ term$$

7(b). If $\tan(\alpha+i\beta) = x+iy$, then prove that $x^2+y^2+2x\cot 2\alpha =$ 5
1 & $x^2+y^2-2y\coth 2\beta = -1$