International Islamic University Chittagong (IIUC)
Department of Computer Science and Engineering (CSE)
B. Sc. in CSE, Semester Final Examination, Autumn-2018
Course Code: MATH-2307, Course Title: Mathematics-III

Time: 2:30 hours **Marks: 50**

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[Answer any two questions from Group-A. and any three questions from Group-B. Separate answer script must be used for separate group. Figures in the right margin indicates full marks]

Group – A

1. Examine the Eigen Decomposition for the matrix, $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ **10**

2. a) If $\emptyset(x, y, z) = xy^2z$ and $\vec{A} = xz\hat{\imath} - xy^2\hat{\jmath} + yz^2\hat{k}$, then find $\frac{\partial^3}{\partial x^2\,\partial z}(\emptyset\vec{A})$ at the point $(2, -1, 1)$ **5**

b) A particle moves along a curve whose parametric equations are $x = t^2 + 1$, $y = 4t - 3$ & $z = 2t^2 - 6t$, where t is the time **5**

(i) Determine the unit tangent vector at any time *t*

(ii) Find the unit tangent vector at *t=2*

3. a) If $\boldsymbol{A} = x^2yz\,\hat{\imath} - 2xz^3\hat{\jmath} + xz^2\hat{k}$ and $\boldsymbol{B} = 2z\,\hat{\imath} + y\hat{\jmath} - x^2\hat{k}$ then find $\frac{\partial^2}{\partial x\,\partial y}(\mathbf{A} \times \boldsymbol{B})$ at (1, 0, -2) **5**

b) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 2$ in the direction $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ **5**

Group-B

4. a) Define Gradient, Divergence and Curl. Prove that, $\nabla^2(ln\, r) = \frac{1}{r^2}$ **5**

b) Prove that, $curl(\emptyset\vec{F}) = grad\emptyset \times \vec{F}$; where $\vec{F}$ is irrotational and $\emptyset(x, y, z)$ is a scalar **5**

5. a) Evaluate the line integral $\int_C xy\,dx$ along the curve C that the portion of $x^2 + y^2 = 1$ in the first quadrant. **5**

b) Find work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$ & $z = t^3$ from $t = 1$ to $t = 2$ **5**

6. State Green's theorem. Verify Green's theorem in the plane for $\oint_c \{(xy + y^2)\,dx + x^2dy\}$ where C is the close curve of the region bounded by $y = x$ and $y = x^2$ **10**

7. a) Evaluate $\iint \vec{F}.\hat{\eta}\,ds$ Where, $\vec{F} = z\hat{\imath} + x\hat{\jmath} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ to $z = 5$ **6**

b) State Divergence theorem. Evaluate $\iint \vec{F}.\hat{\eta}\,ds$ by using divergence theorem, Where $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and S is the surface of the cube bounded by the planes, $x = 0$, $x = 1$; $y = 0, y = 3$; $z = 0$, $z = 2$ **4**